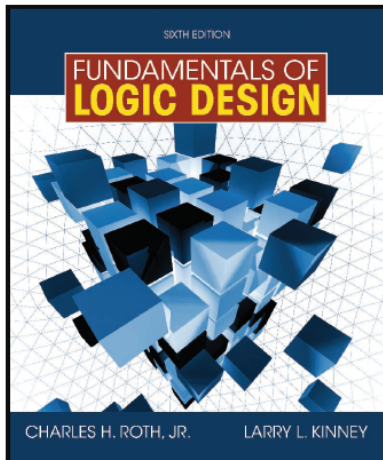


SLIDES FOR CHAPTER 5

KARNAUGH MAPS



This chapter in the book includes:

- Objectives
 - Study Guide
 - 5.1 Minimum Forms of Switching Functions
 - 5.2 Two- and Three-Variable Karnaugh Maps
 - 5.3 Four-Variable Karnaugh Maps
 - 5.4 Determination of Minimum Expressions
 - 5.5 Five-Variable Karnaugh Maps
 - 5.6 Other Uses of Karnaugh Maps
 - 5.7 Other Forms of Karnaugh Maps
- Programmed Exercises
Problems

**Click the mouse to move to the next page.
Use the ESC key to exit this chapter.**

Karnaugh Maps

Switching functions can generally be simplified by using the algebraic techniques described in Unit 3. However, two problems arise when algebraic procedures are used:

1. The procedures are difficult to apply in a systematic way.
2. It is difficult to tell when you have arrived at a minimum solution.

The Karnaugh map method is generally faster and easier to apply than other simplification methods.

Chapter 5 (p. 127)

Minimum Forms of Switching Functions

Given a minterm expansion, the minimum sum-of-products format can often be obtained by the following procedure:

1. Combine terms by using $XY' + XY = X$. Do this repeatedly to eliminate as many literals as possible. A given term may be used more than once because $X + X = X$.
2. Eliminate redundant terms by using the consensus theorem or other theorems.

Unfortunately, the result of this procedure may depend on the order in which the terms are combined or eliminated so that the final expression obtained is not necessarily minimum.

Section 5.1 (p. 128)

Example

Find a minimum sum-of-products expression for

$$F(a, b, c) = \sum m (0, 1, 2, 5, 6, 7)$$

$$\begin{aligned} F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\ &= a'b' + b'c + bc' + ab \end{aligned} \quad (5-1)$$

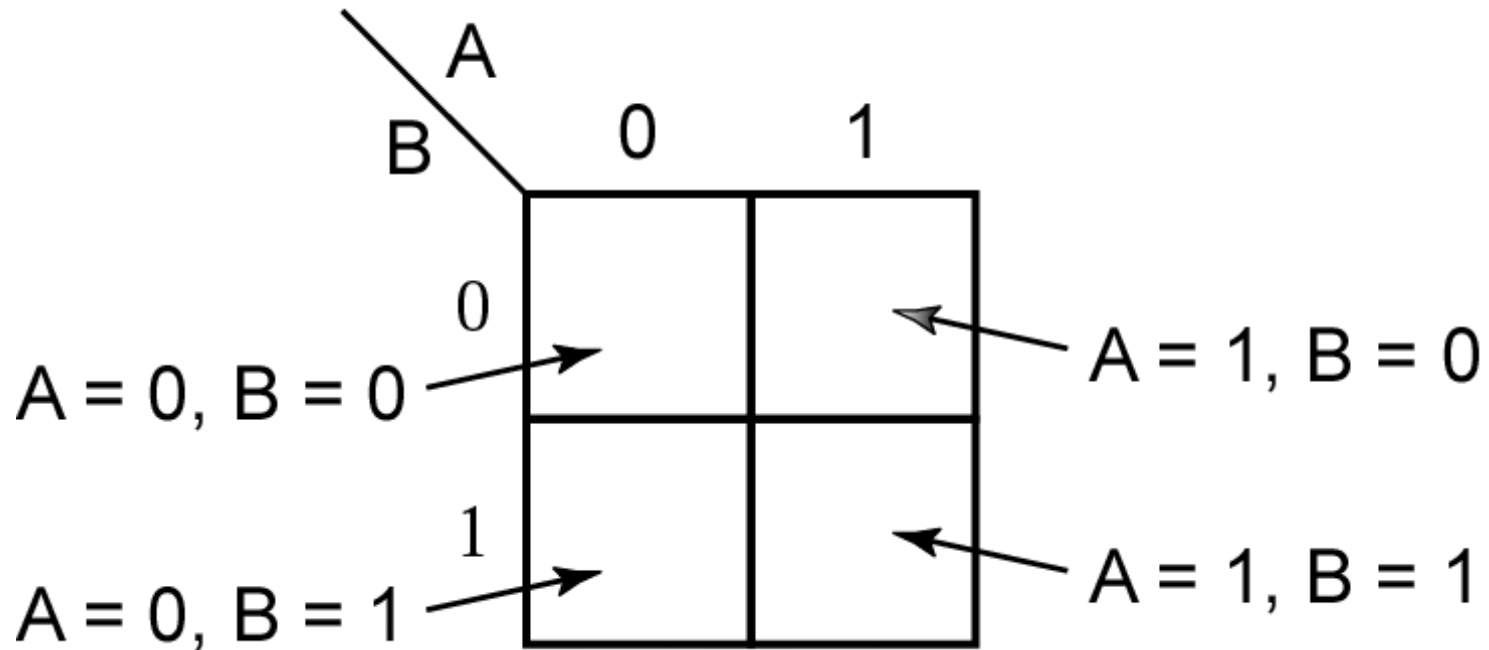
None of the terms in the above expression can be eliminated by consensus. However, combining terms in a different way leads directly to a minimum sum of products:

$$\begin{aligned} F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\ &= a'b' + bc' + ac \end{aligned} \quad (5-2)$$

Section 5.1 (p. 128)

Two- and Three- Variable Karnaugh Maps

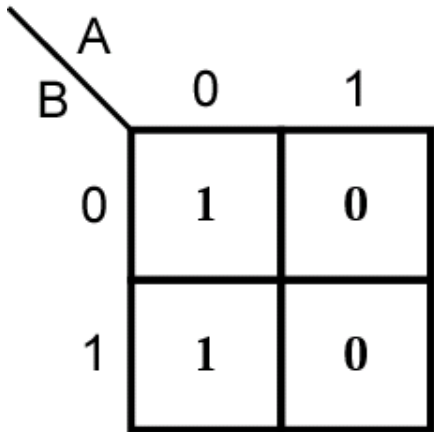
Just like a truth table, the Karnaugh map of a function specifies the value of the function for every combination of values of the independent variables.



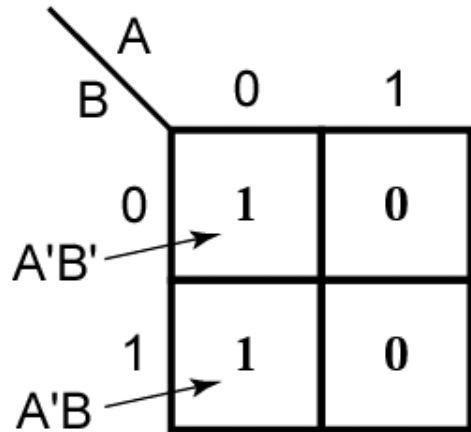
Section 5.1 (p. 129)

(a)

<i>A</i>	<i>B</i>	<i>F</i>
0	0	1
0	1	1
1	0	0
1	1	0

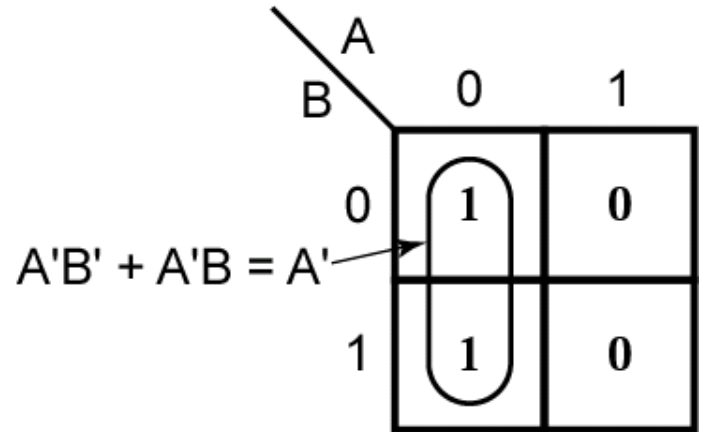


(b)



$$F = A'B' + A'B$$

(c)



$$A'B' + A'B = A'$$

$$F = A'$$

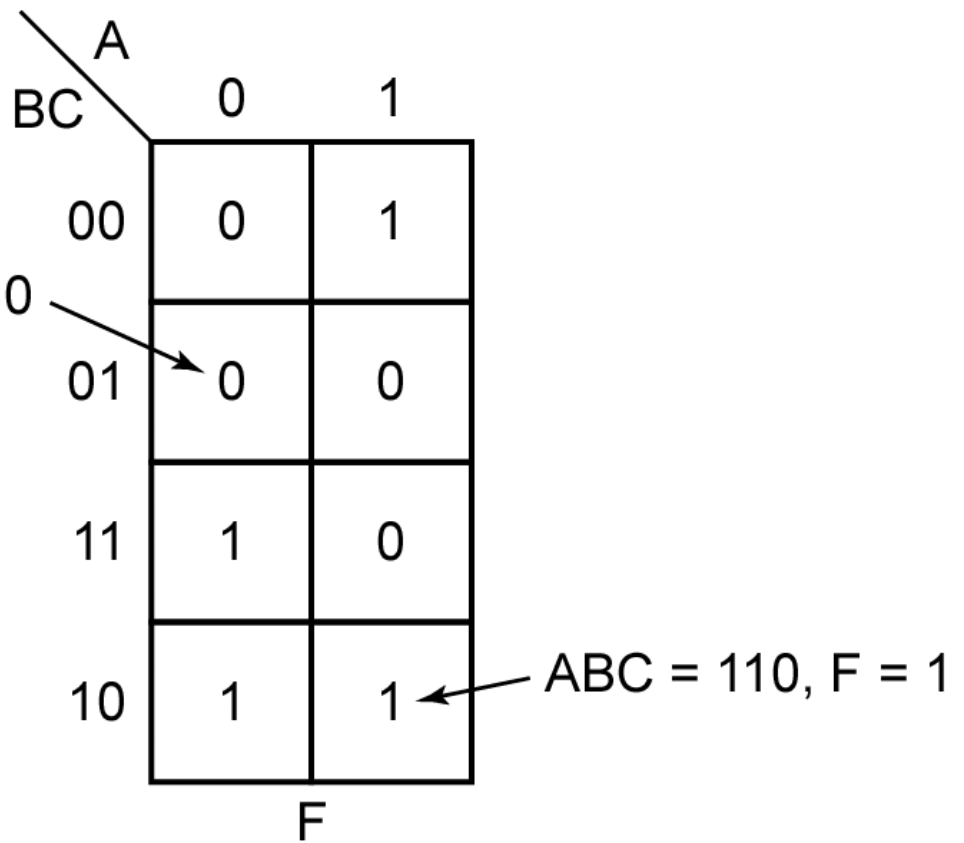
(d)

Figure 5-1

<i>A</i>	<i>B</i>	<i>C</i>	<i>F</i>
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

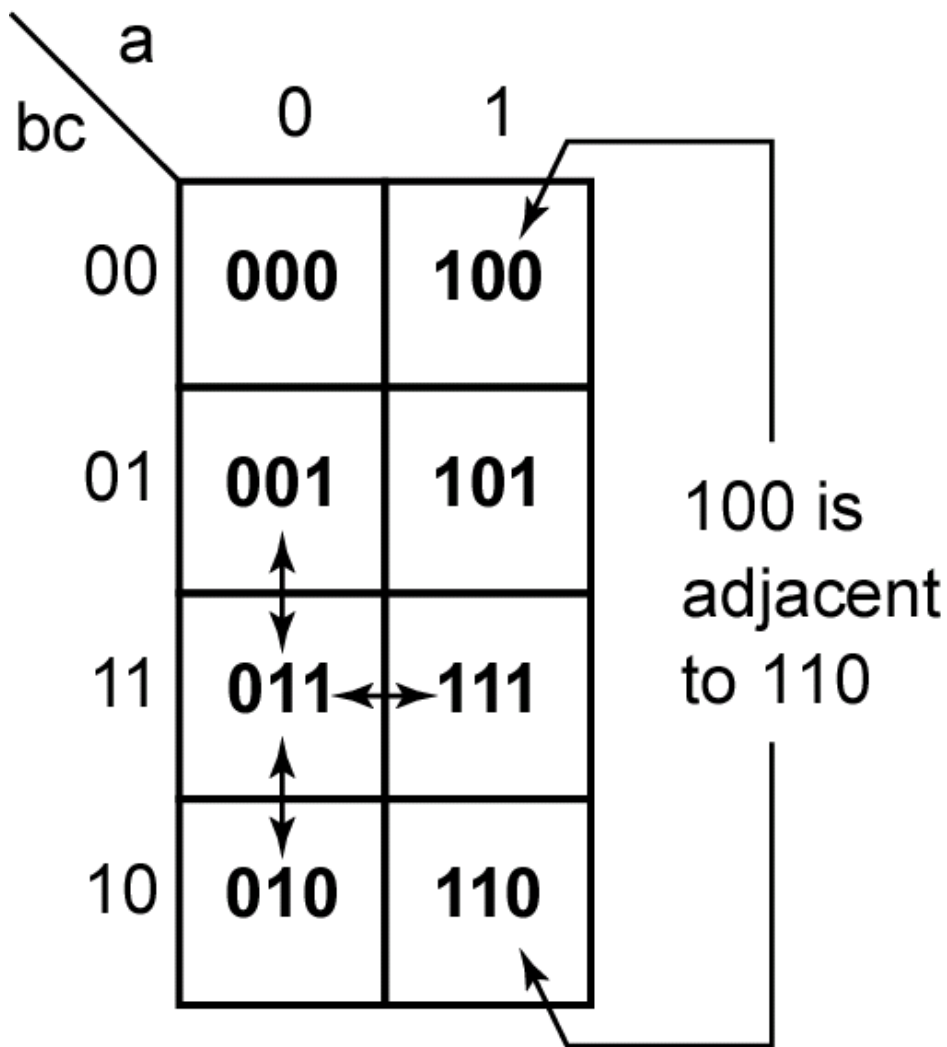
(a)

ABC = 001, F = 0

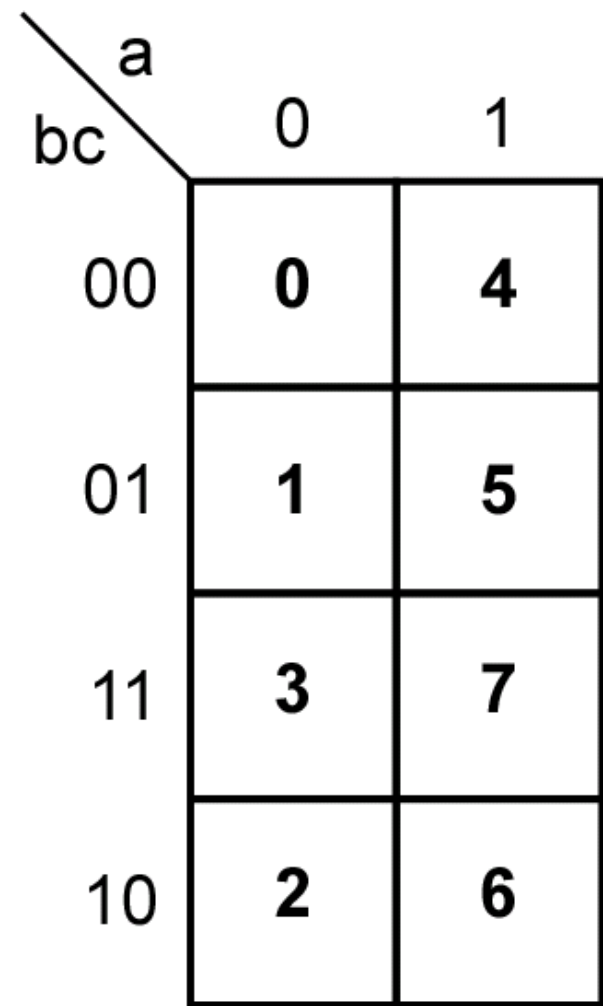


(b)

Figure 5-2: Karnaugh Map for Three-Variable Function



(a) Binary notation



(b) Decimal notation

Figure 5-3: Location of Minterms on a Three-Variable Karnaugh Map

		a	
		0	1
bc	00	0 0	0 4
	01	1 1	1 5
	11	1 3	0 7
	10	0 2	0 6

Figure 5-4: Karnaugh Map of $F(a, b, c) = \Sigma m(1, 3, 5) = \Pi M(0, 2, 4, 6, 7)$

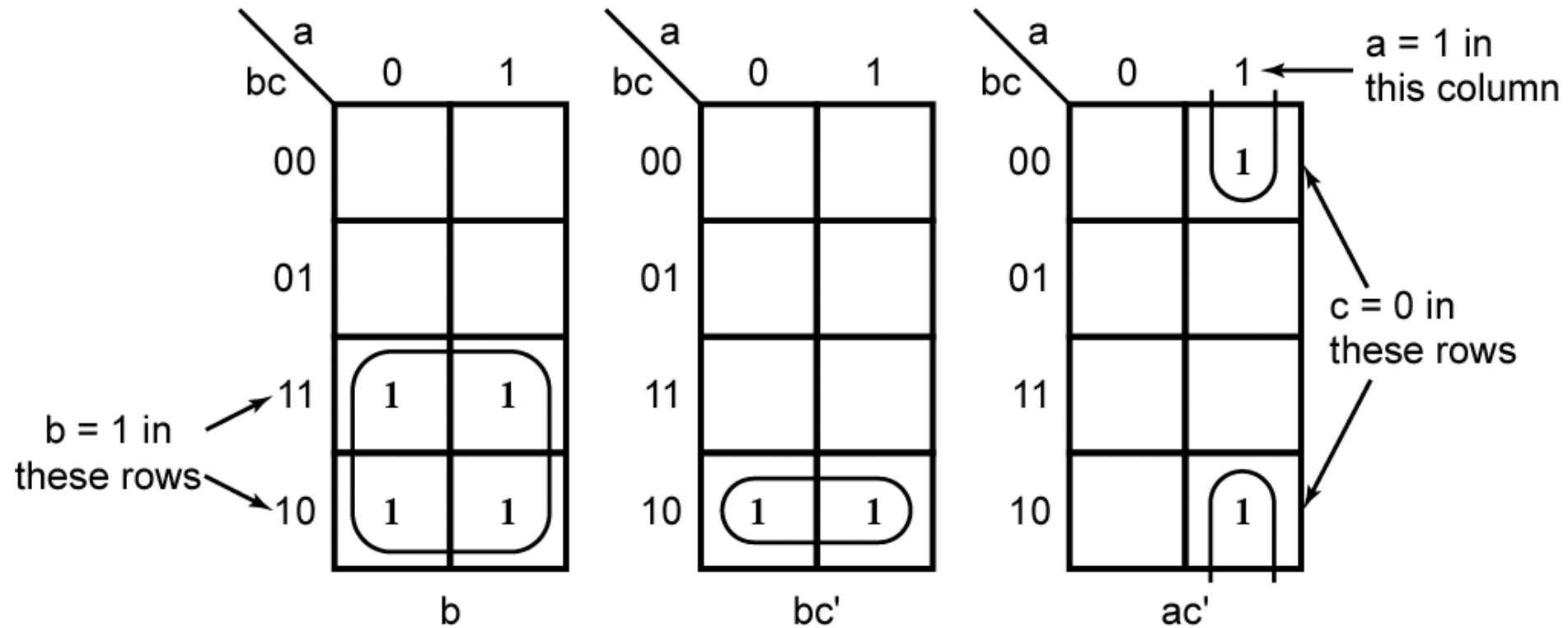
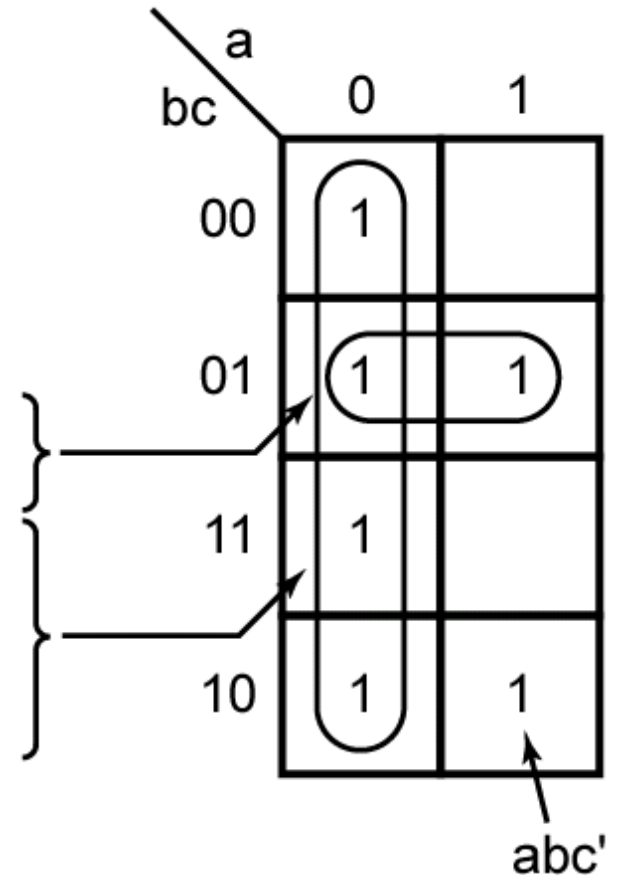


Figure 5-5: Karnaugh Maps for Product Terms

$$f(a,b,c) = abc' + b'c + a'$$

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1's in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1's in all the squares of the $a = 0$ column of the map. (Note: Since there already is a 1 in the $abc = 001$ square, we do not have to place a second 1 there because $x + x = x$.)



Section 5.2 (p. 131)

	a		
		bc	
			0 1
00			
01		1	1
11		1	
10			

$$F = \sum m(1, 3, 5)$$

(a) Plot of minterms

$$T_1 = a'b'c + a'bc$$

$$= a'c$$

	a		
		bc	
			0 1
00			
01		1	1
11		1	
10			

$$F = a'c + b'c$$

(b) Simplified form of F

$$T_2 = a'b'c + ab'c$$

$$= b'c$$

Figure 5-6: Simplification of a Three-Variable Function

The map for the complement of F is formed by replacing 0's with 1's and 1's with 0's on the map of F .

$$T_1 = b'c' + bc' = c'$$

$$F' = c' + ab$$

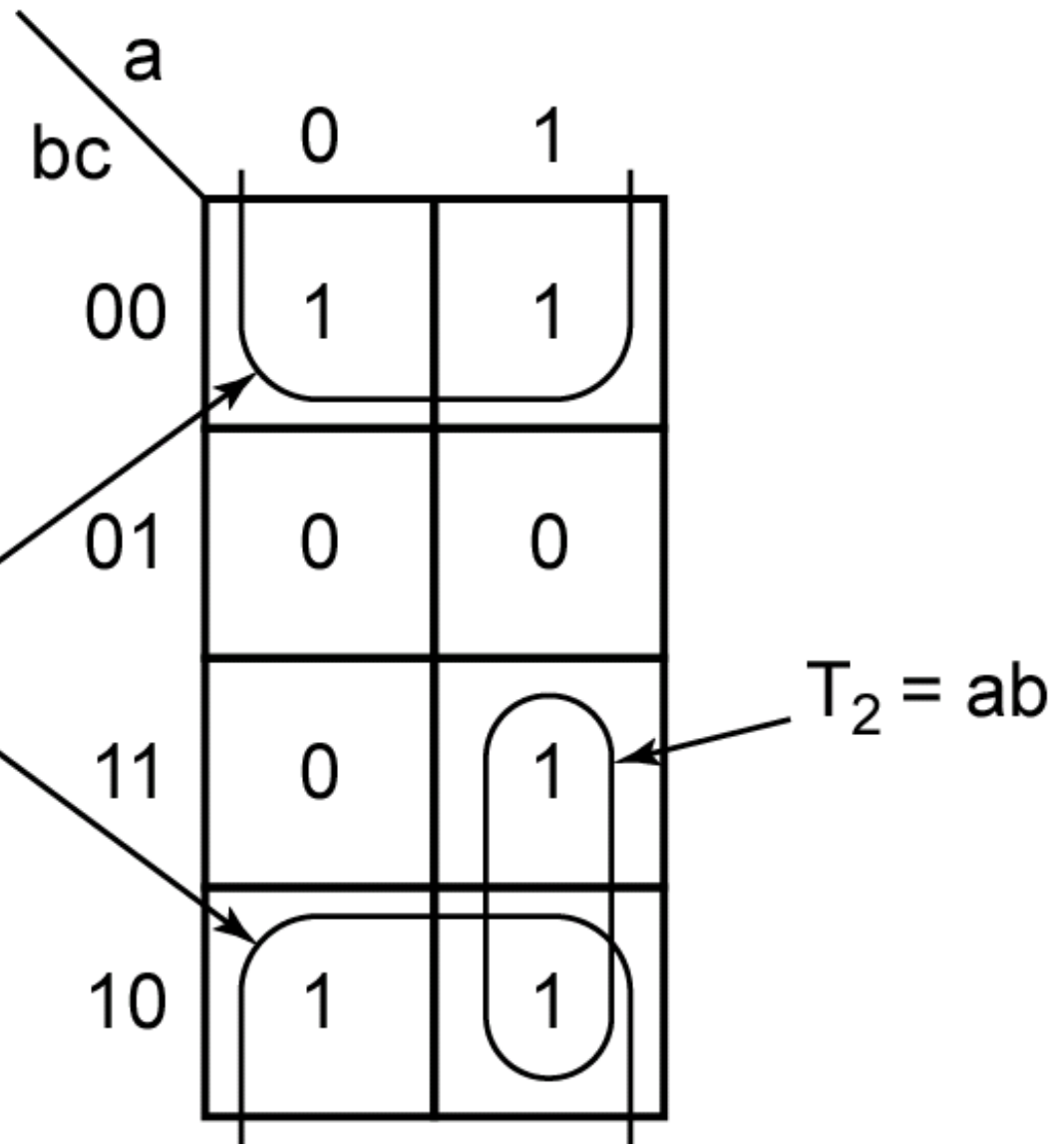


Figure 5-7: Complement of Map in Figure 5-6a

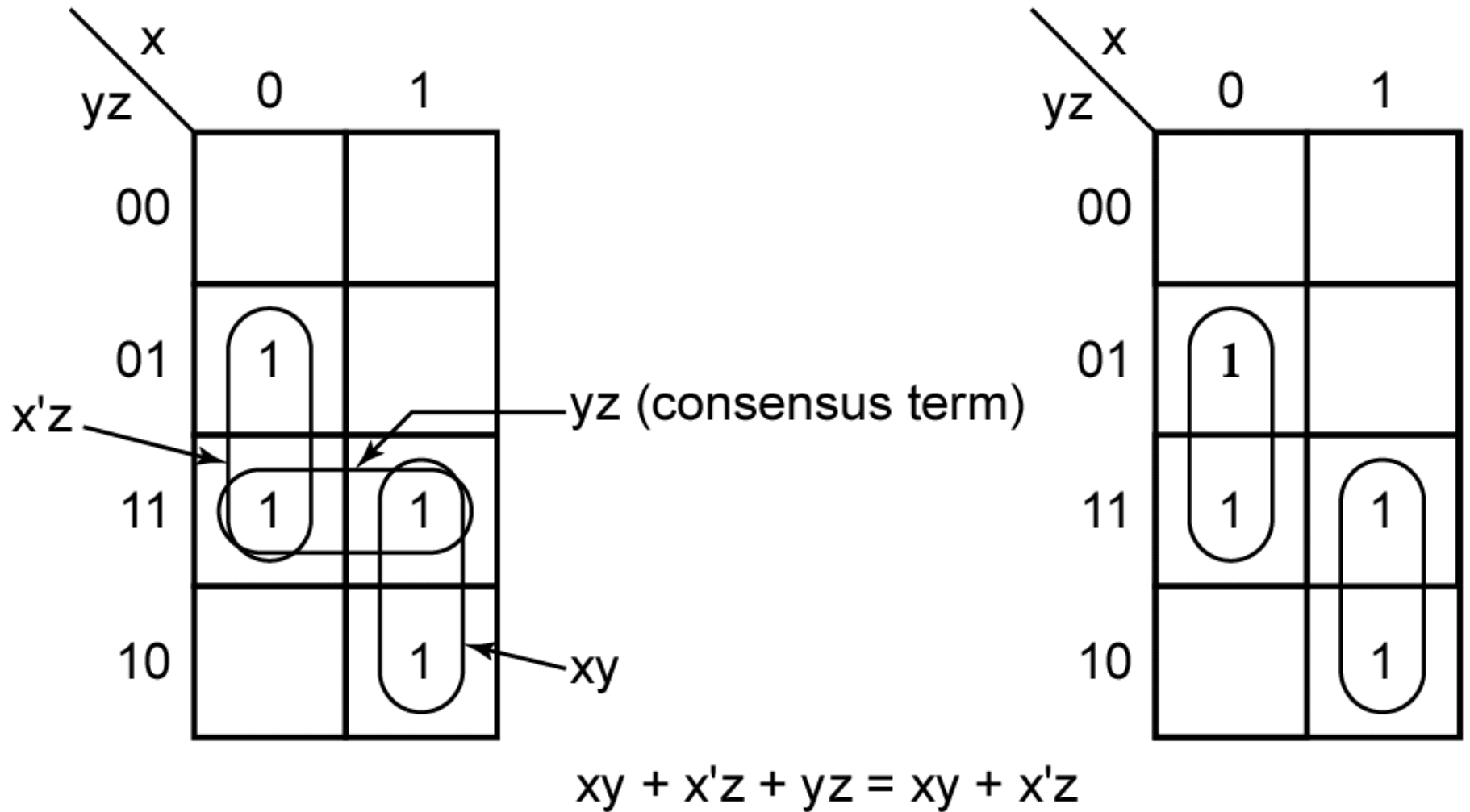
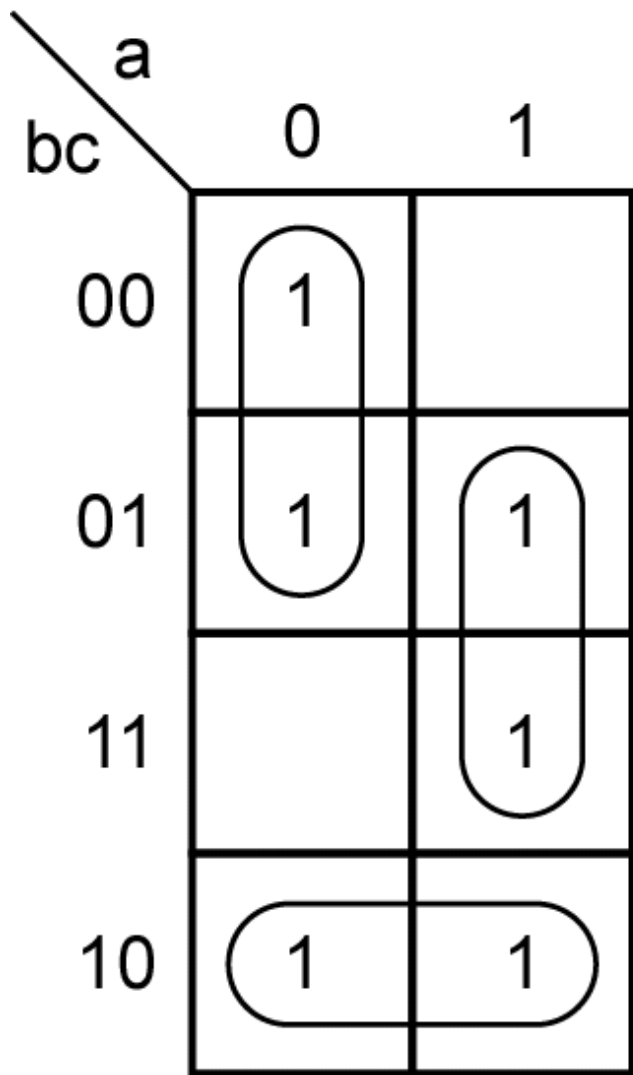
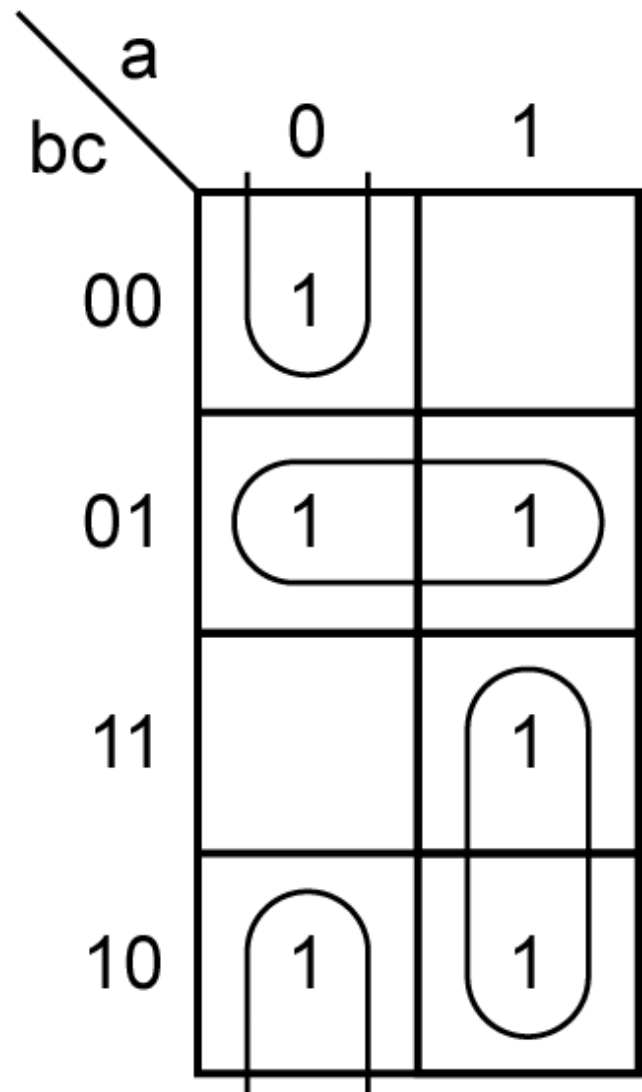


Figure 5-8: Karnaugh Maps Which Illustrate the Consensus Theorem



$$F = a'b' + bc' + ac$$



$$F = a'c' + b'c + ab$$

Figure 5-9: Function with Two Minimal Forms

Each minterm is located adjacent to the four terms with which it can combine. For example, m_5 (0101) could combine with m_1 (0001), m_4 (0100), m_7 (0111), or m_{13} (1101).

		AB			
		00	01	11	10
CD	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

Figure 5-10: Location of Minterms on Four-Variable Karnaugh Map

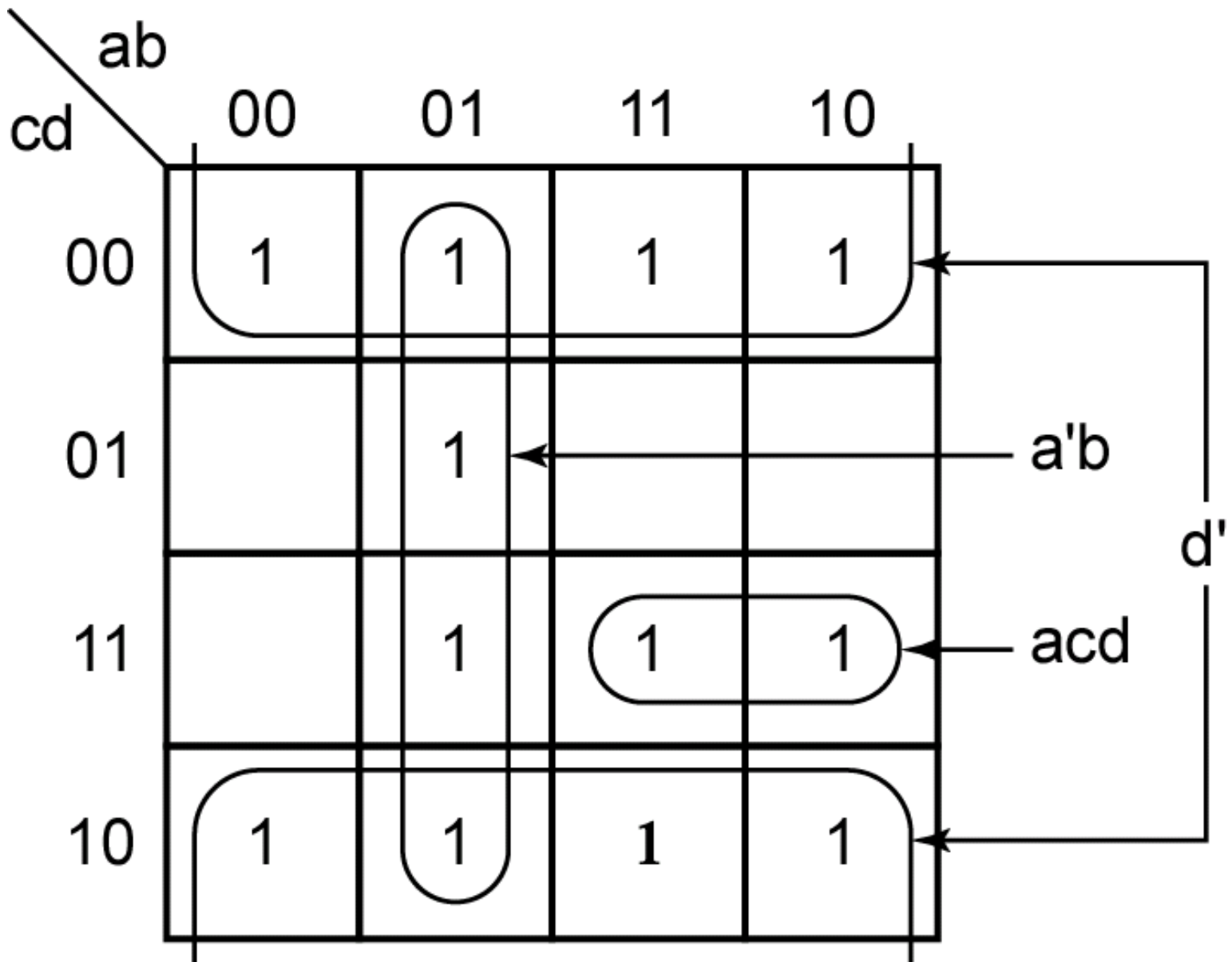
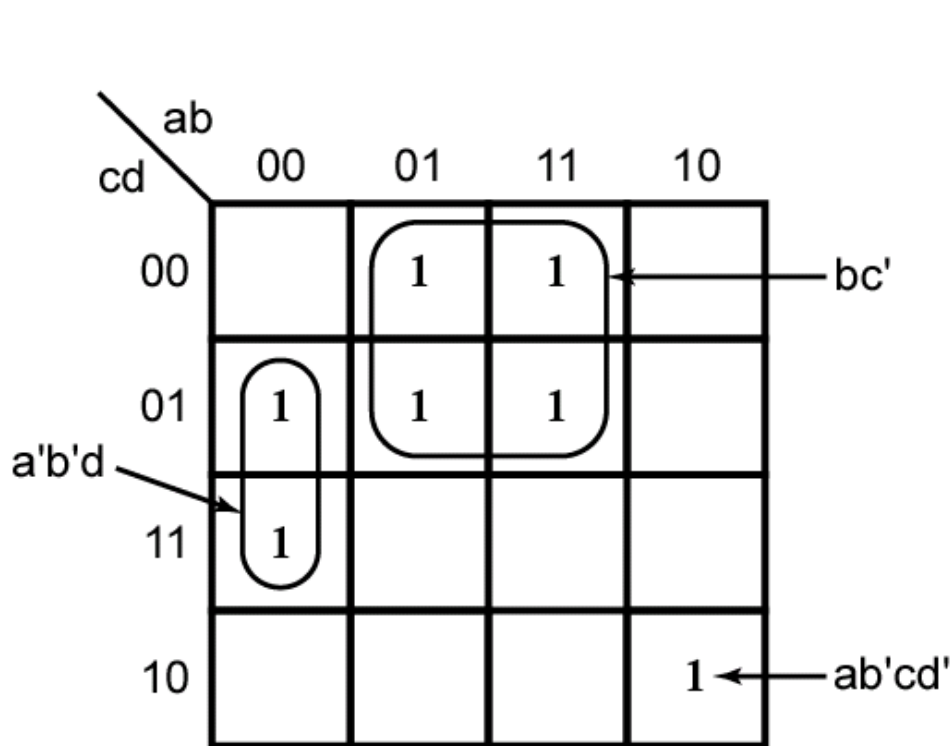
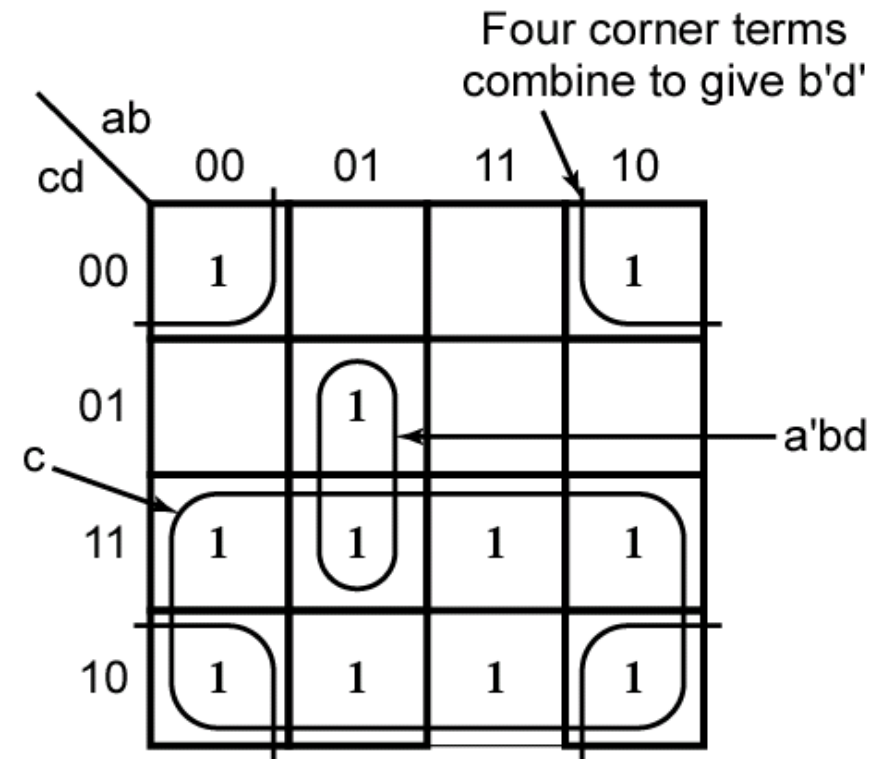


Figure 5-11: Plot of $acd + a'b + d'$



(a)



(b)

Figure 5-12: Simplification of Four-Variable Functions

		ab			
cd		00	01	11	10
00				X	
01		1	1	X	1
11		1	1		
10			X		

$$f = \sum m(1, 3, 5, 7, 9) + \sum d(6, 12, 13)$$

$$= a'd + c'd$$

Figure 5-13: Simplification of an Incompletely Specified Function

Example

Use a four variable Karnaugh map to find the minimum product of sums for f :

$$f = x'z' + wyz + w'y'z' + x'y$$

First, we will plot the 1's on a Karnaugh map.

Section 5.3 (p. 136)

Then, from the 0's we get:

$$f' = y'z + wxz' + w'xy$$

Finally, we can complement f' to get a minimum product of sums of f :

$$f = (y + z')(w' + x' + z)(w + x' + y')$$

A 4x4 Karnaugh map for variables w, x, y, z. The columns are labeled wx (00, 01, 11, 10) and the rows are labeled yz (00, 01, 11, 10). The map contains 1s in cells (wx, yz) = (00, 00), (01, 00), (10, 00), (00, 11), (10, 11), (00, 10), and (10, 10). It contains 0s in cells (11, 00), (00, 01), (01, 01), (11, 01), (10, 01), (01, 11), (11, 11), (01, 10), (11, 10), and (10, 10). Groupings for 0s are shown: a vertical group of two 0s in the wx=11 column (yz=00, 01); a horizontal group of four 0s in the yz=01 row; a vertical group of two 0s in the wx=01 column (yz=11, 10); and a vertical group of two 0s in the wx=11 column (yz=11, 10).

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1

Figure 5-14

Implicants and Prime Implicants

Any single 1 or any group of 1's which can be combined together on a map of the function F represents a product term which is called an *implicant* of F .

A product term implicant is called a *prime implicant* if it cannot be combined with another term to eliminate a variable.

Section 5.4 (p. 136)

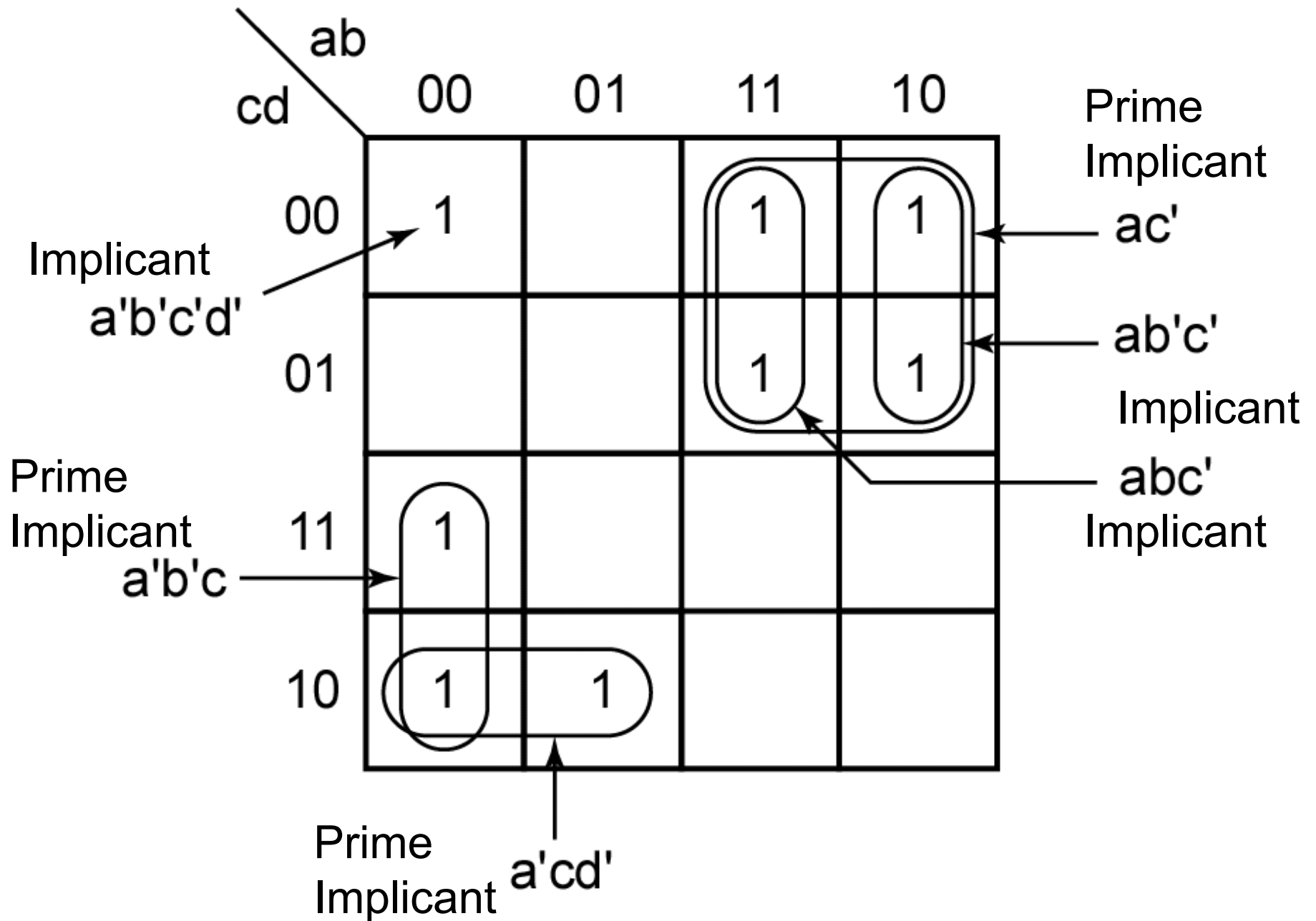


Figure 5-15

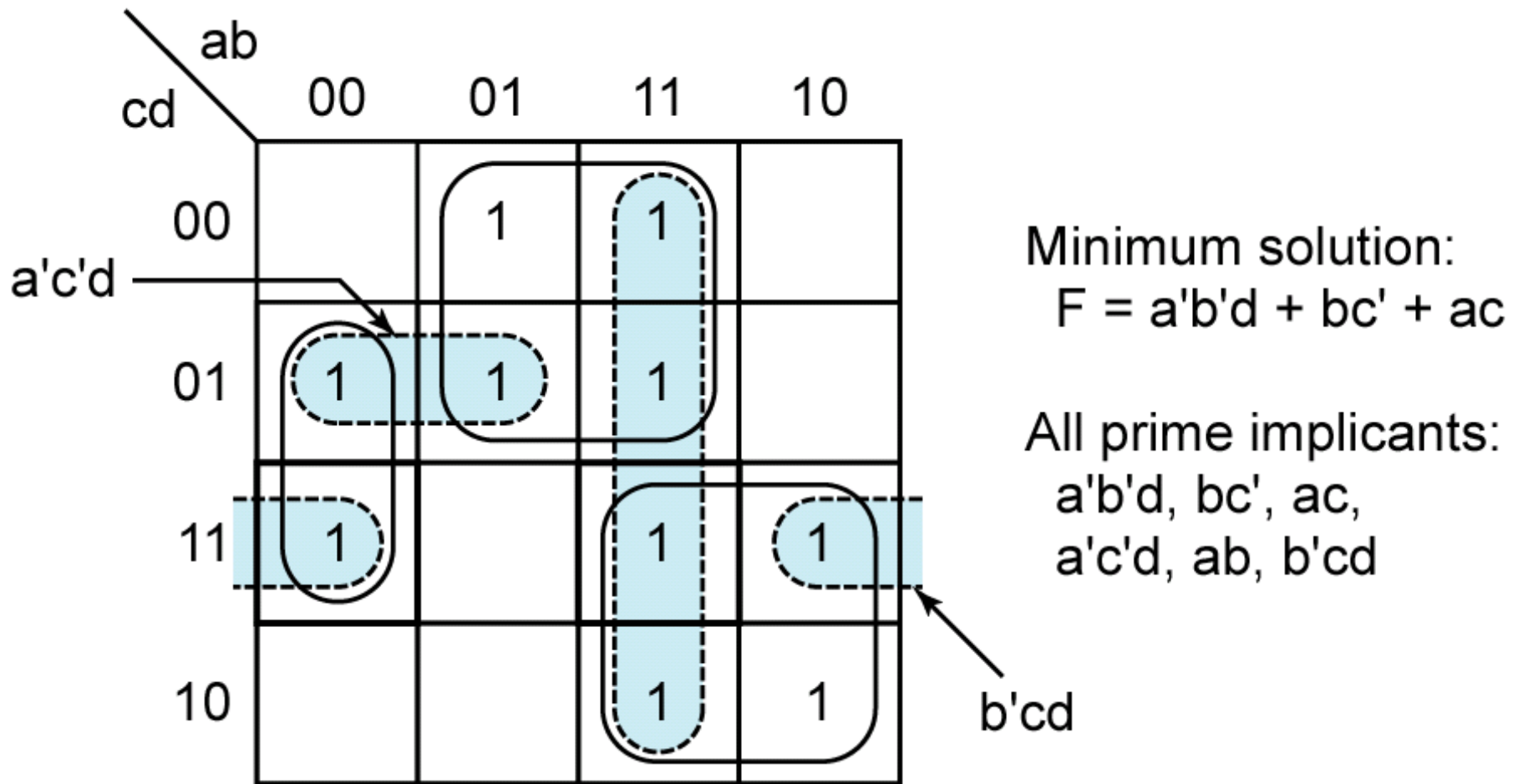
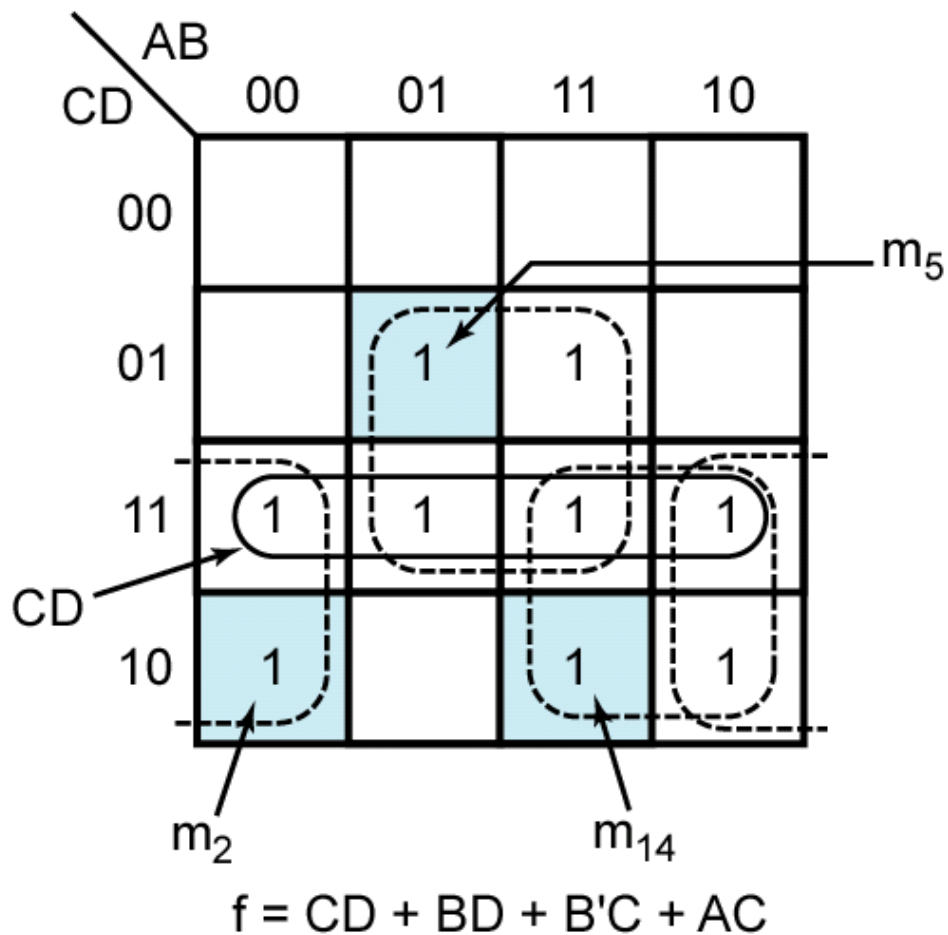
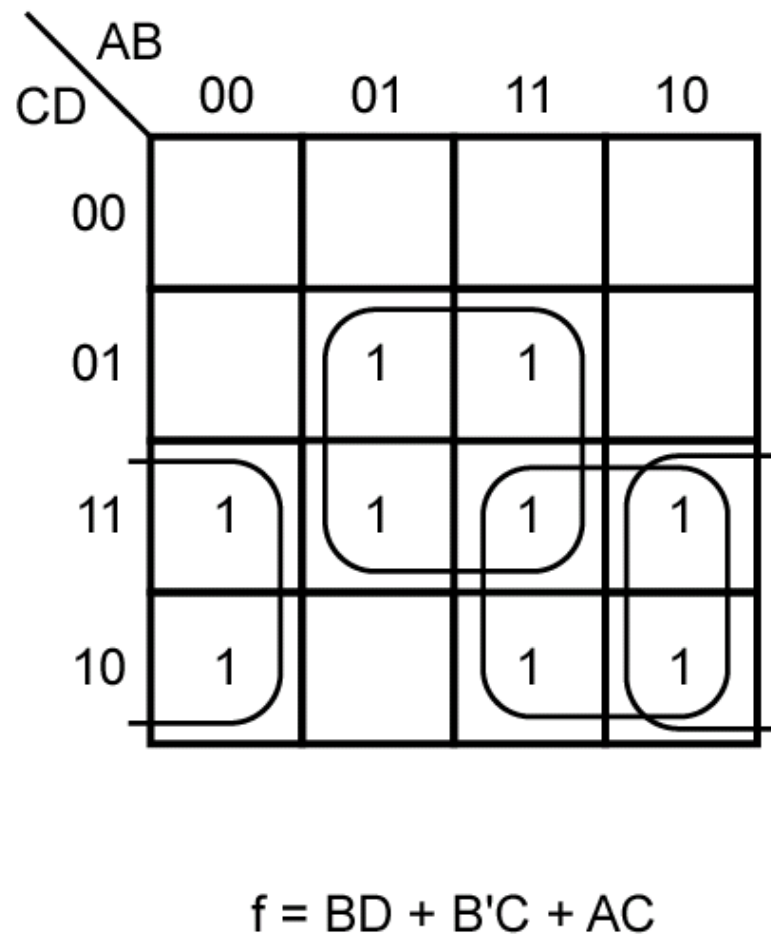


Figure 5-16: Determination of All Prime Implicants



(a)

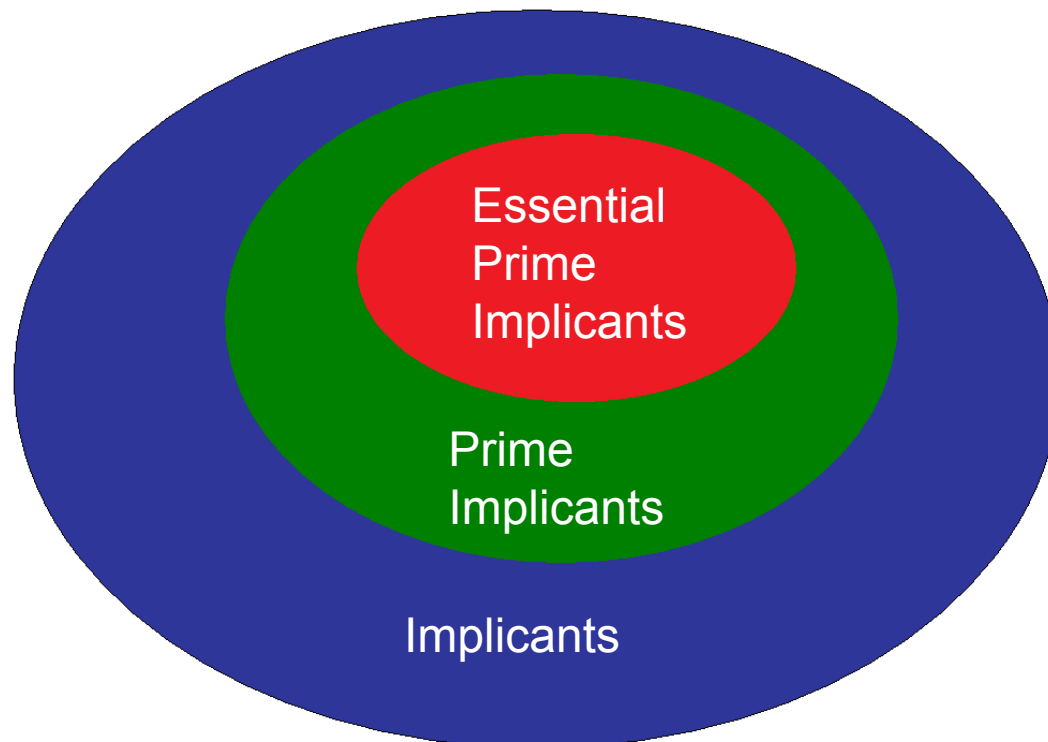


(b)

Figure 5-17

Essential Prime Implicants

If a minterm is covered by only one prime implicant, that prime implicant is said to be *essential*, and it must be included in the minimum sum of products.



Section 5.4 (p. 138)

Note: 1's shaded in blue are covered by only one prime implicant. All other 1's are covered by at least two prime implicants.

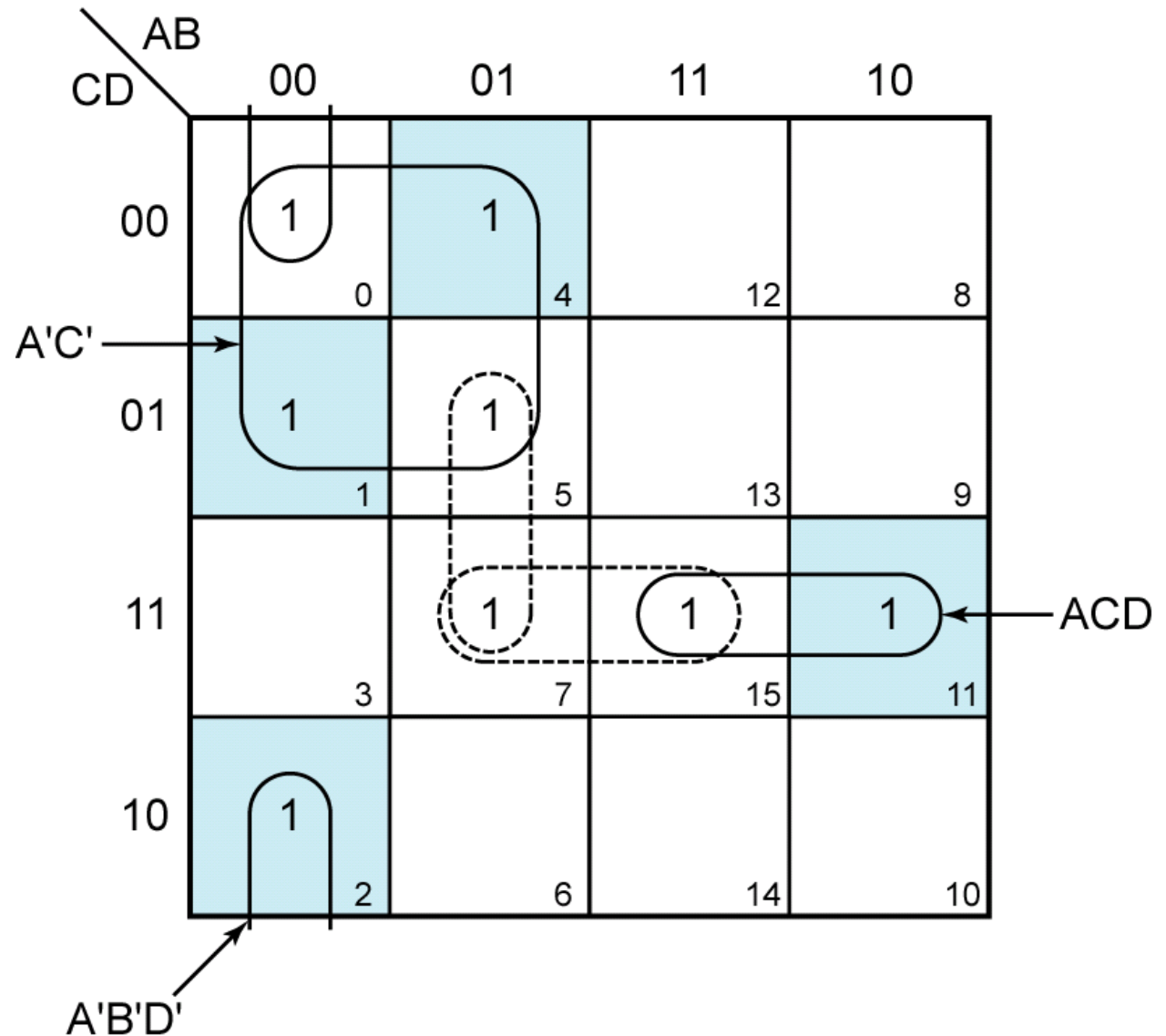


Figure 5-18

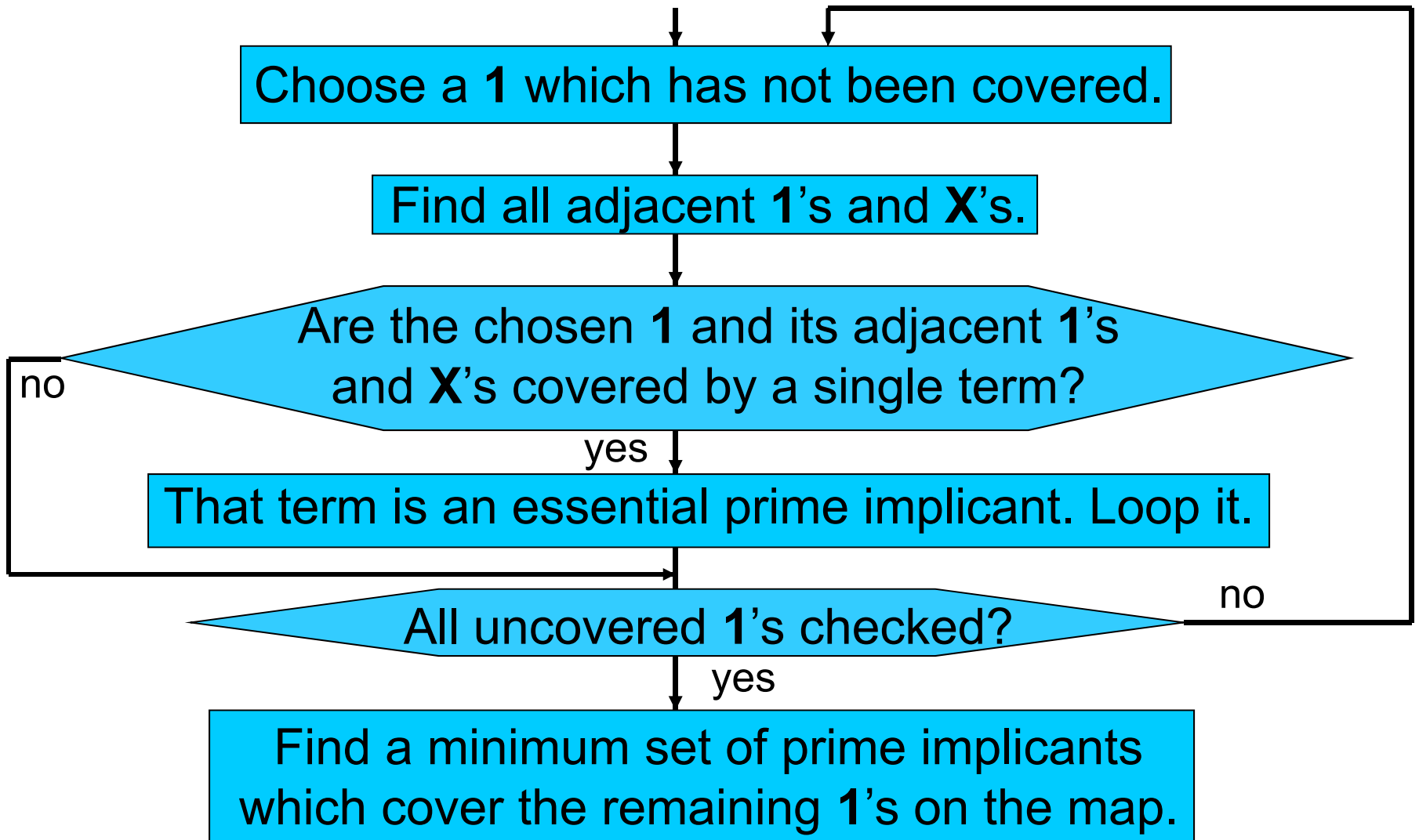
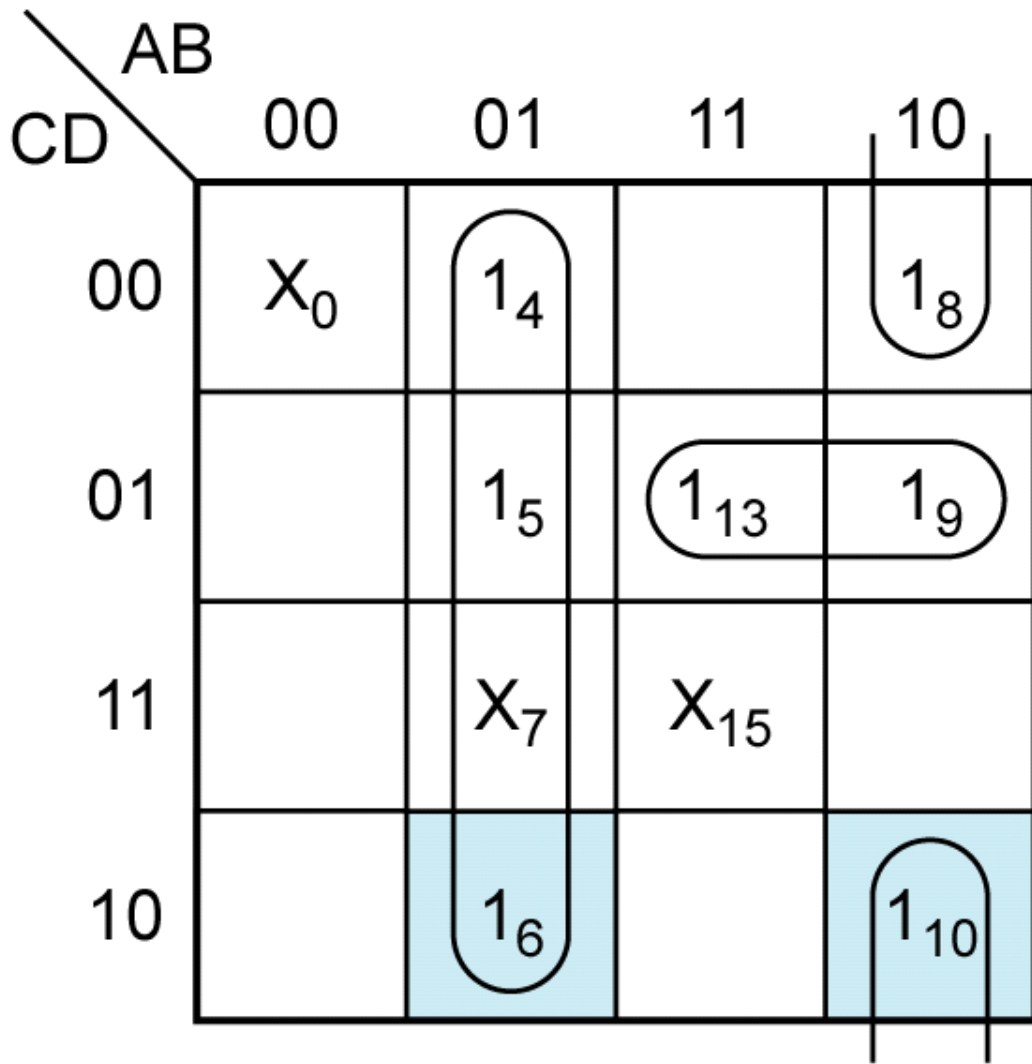


Figure 5-19:
Flowchart for Determining a Minimum Sum of Products
Using a Karnaugh Map



Shaded 1's are covered by only one prime implicant.

Essential prime implicants:

$$A'B, AB'D'$$

Then $AC'D$ covers the remaining 1's.

Figure 5-20

Five-Variable Karnaugh Maps

A five-variable map can be constructed in three dimensions by placing one four-variable map on top of a second one. Terms in the bottom layer are numbered 0 through 15 and corresponding terms in the top layer are numbered 16 through 31, so that the terms in the bottom layer contain A' and those in the top layer contain A .

To represent the map in two dimensions, we will divide each square in a four-variable map by a diagonal line and place terms in the bottom layer below the line and terms in the top layer above the line.

Section 5.5 (p. 141)

Note that terms m_0 and m_{20} do not combine because they are in different layers and different columns (they differ in two variables).

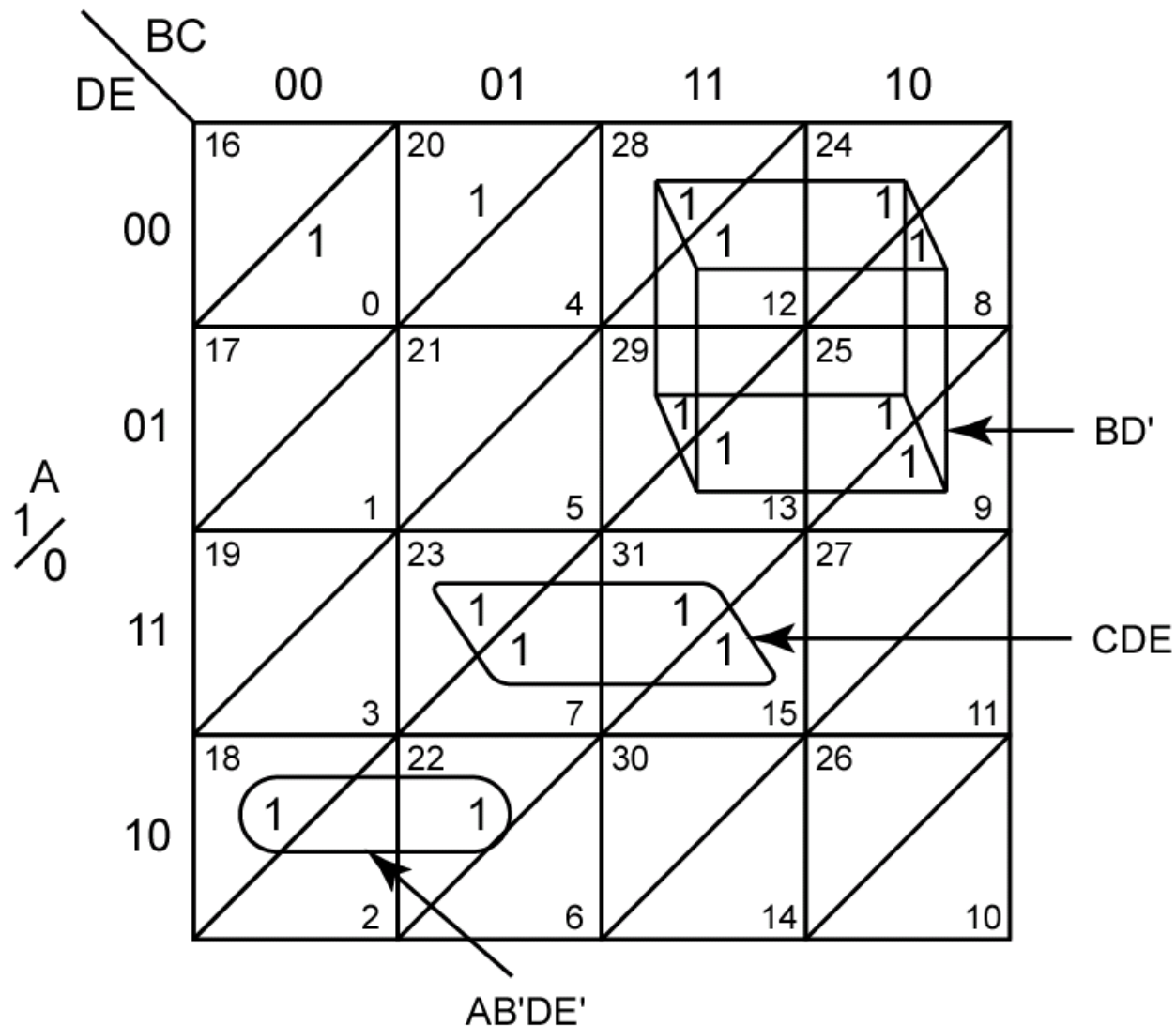


Figure 5-21: A Five-Variable Karnaugh Map

Each term can be adjacent to exactly five other terms: four in the same layer and one in the other layer.

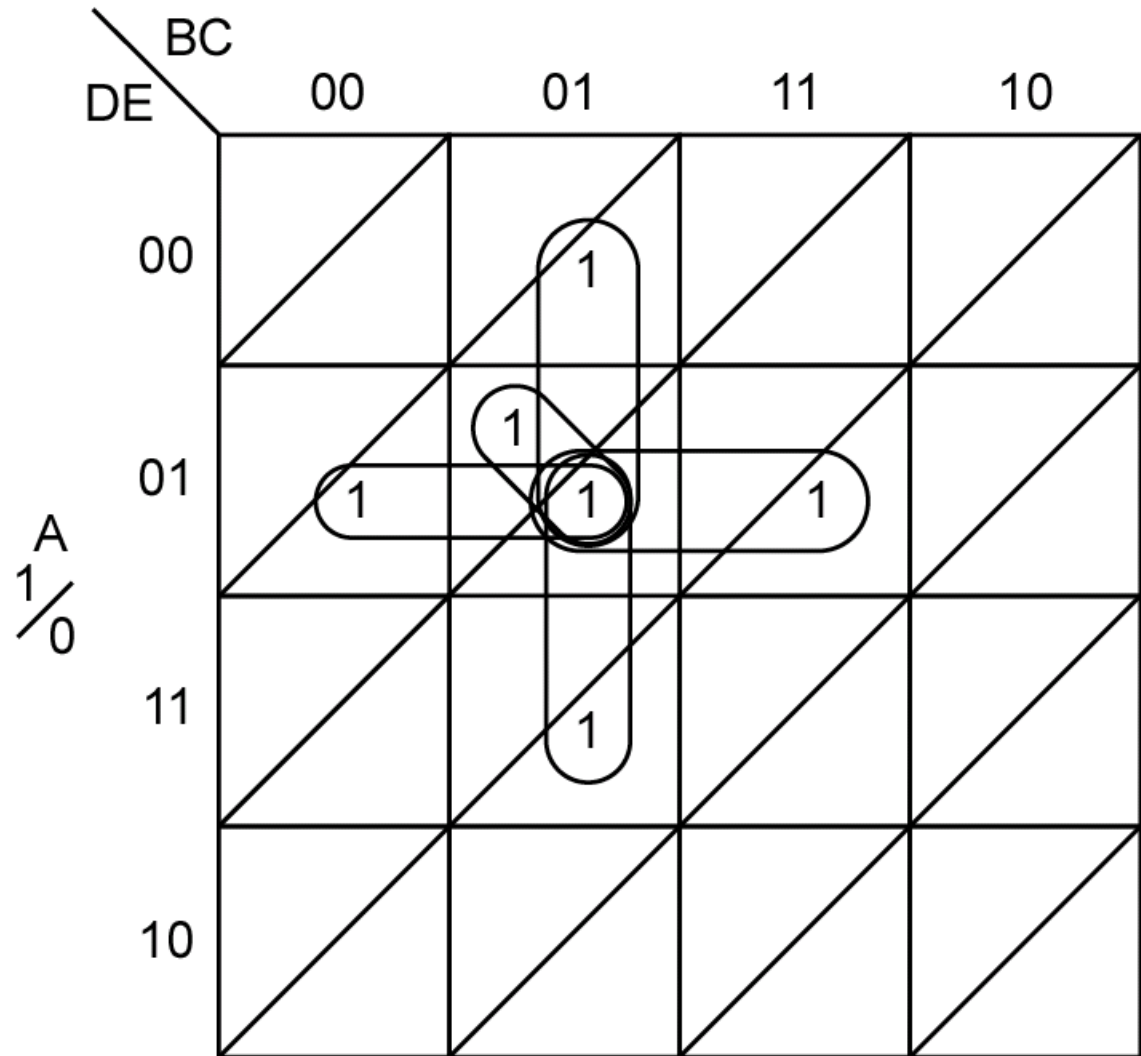


Figure 5-22

When checking for adjacencies, each term should be checked against the five possible adjacent squares.

P_1 and P_2 are essential prime implicants.

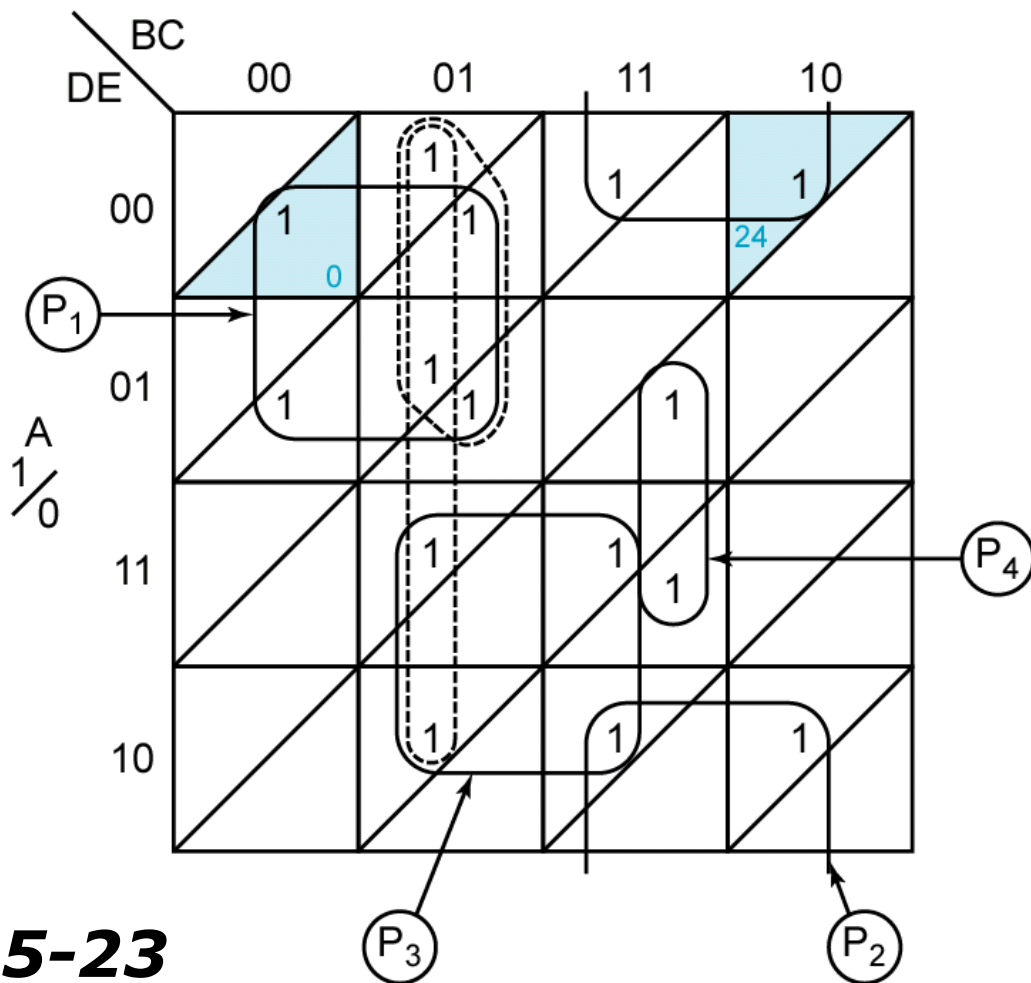
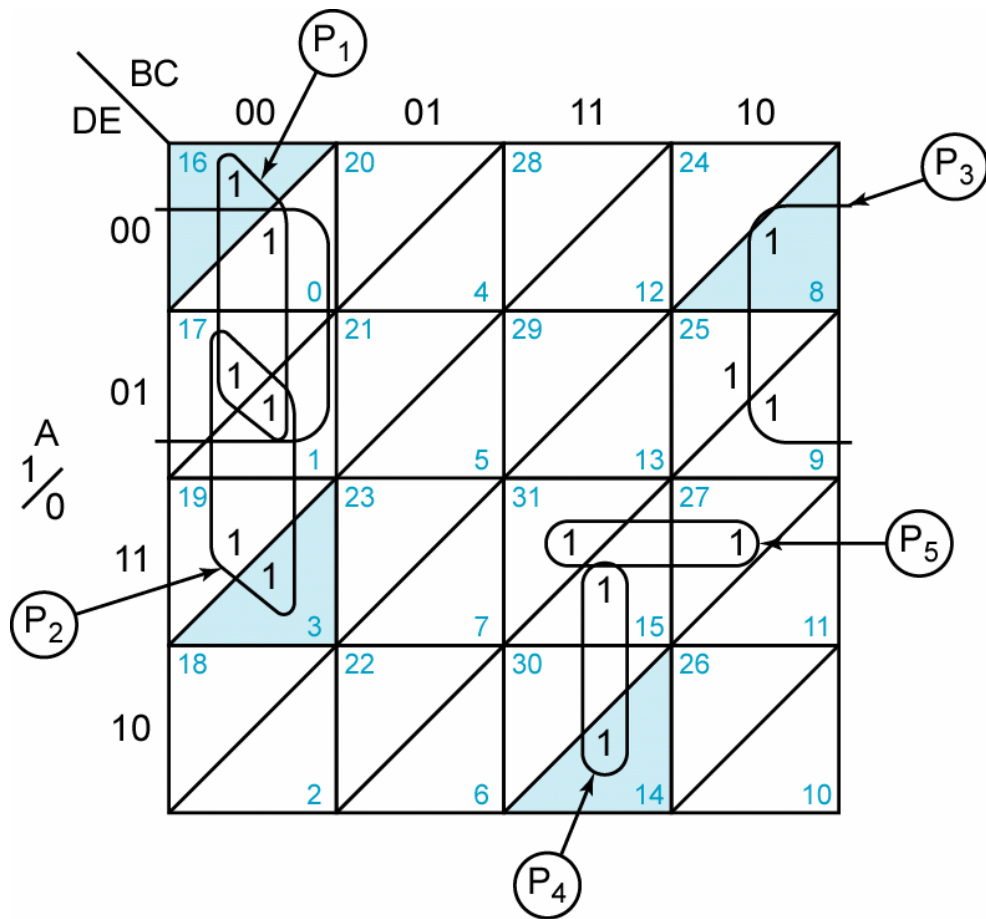


Figure 5-23

$$F = \underbrace{A'B'D'}_{P_1} + \underbrace{ABE'}_{P_2} + \underbrace{ACD}_{P_3} + \underbrace{A'BCE}_{P_4} + \left\{ \begin{array}{c} AB'C \\ \text{or} \\ B'CD' \end{array} \right.$$



P_1 , P_2 , P_3 , and P_4 are essential prime implicants.

$$F(A, B, C, D, E) = \Sigma m(0, 1, 3, 8, 9, 14, 15, 16, 17, 19, 25, 27, 31)$$

$$F = \underbrace{B'C'D'}_{P_1} + \underbrace{B'C'E}_{P_2} + \underbrace{A'C'D'}_{P_3} + \underbrace{A'BCD}_{P_4} + \underbrace{ABDE}_{P_5} + \left. \begin{array}{c} C'D'E \\ \text{or} \\ AC'E \end{array} \right\}$$

Figure 5-24

Using a Karnaugh map to facilitate factoring, we see that the two terms in the first column have $A'B'$ in common; the two terms in the lower right corner have AC in common.

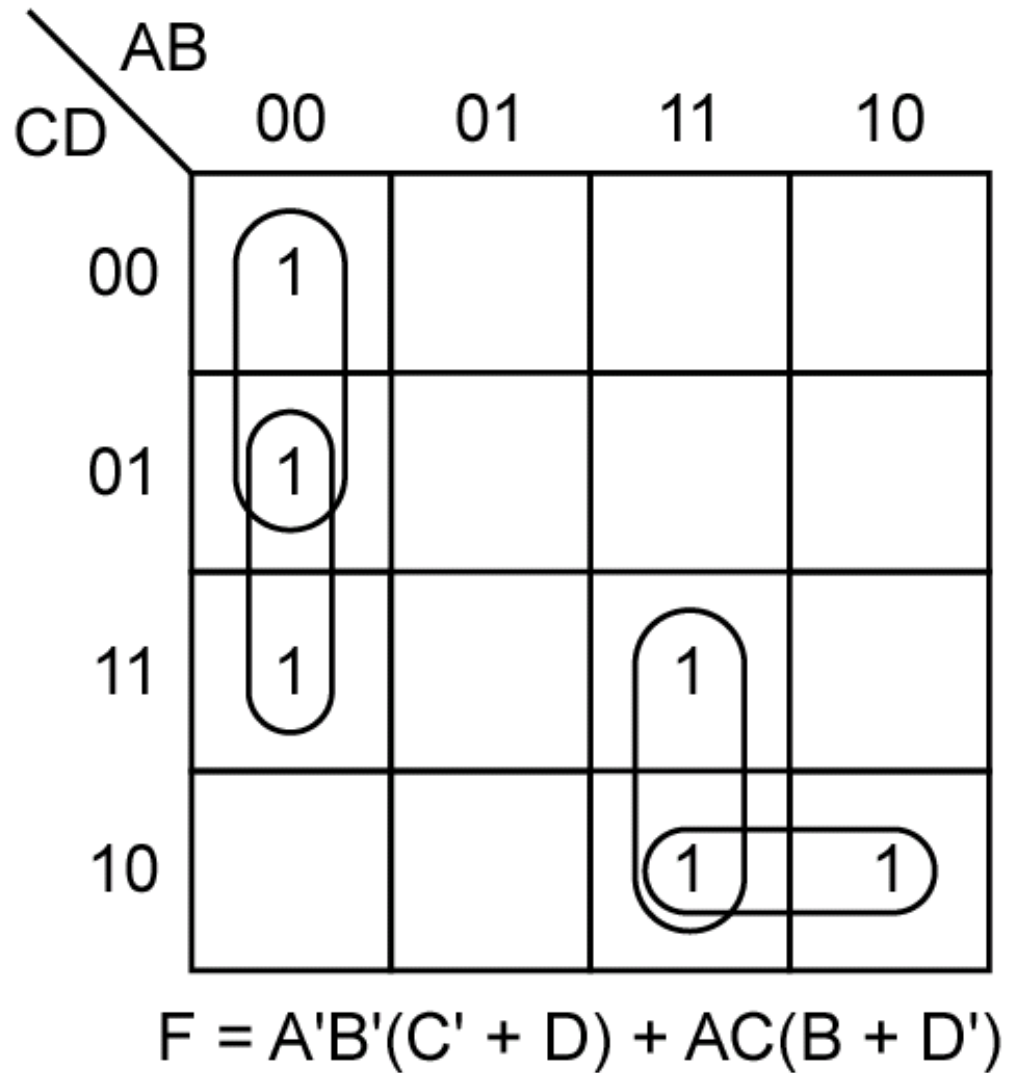



Figure 5-25

$$F = ABCD + B'CDE + A'B' + BCE'$$

We can use a Karnaugh map for guidance in algebraic simplification. From Figure 5-26, we can add the term ACDE by the consensus theorem and then eliminate ABCD and B'CDE.

$$F = \underbrace{ABCD + B'CDE}_{\text{---}} + A'B' + BCE' + ACDE$$


$$F = A'B' + BCE' + ACDE$$

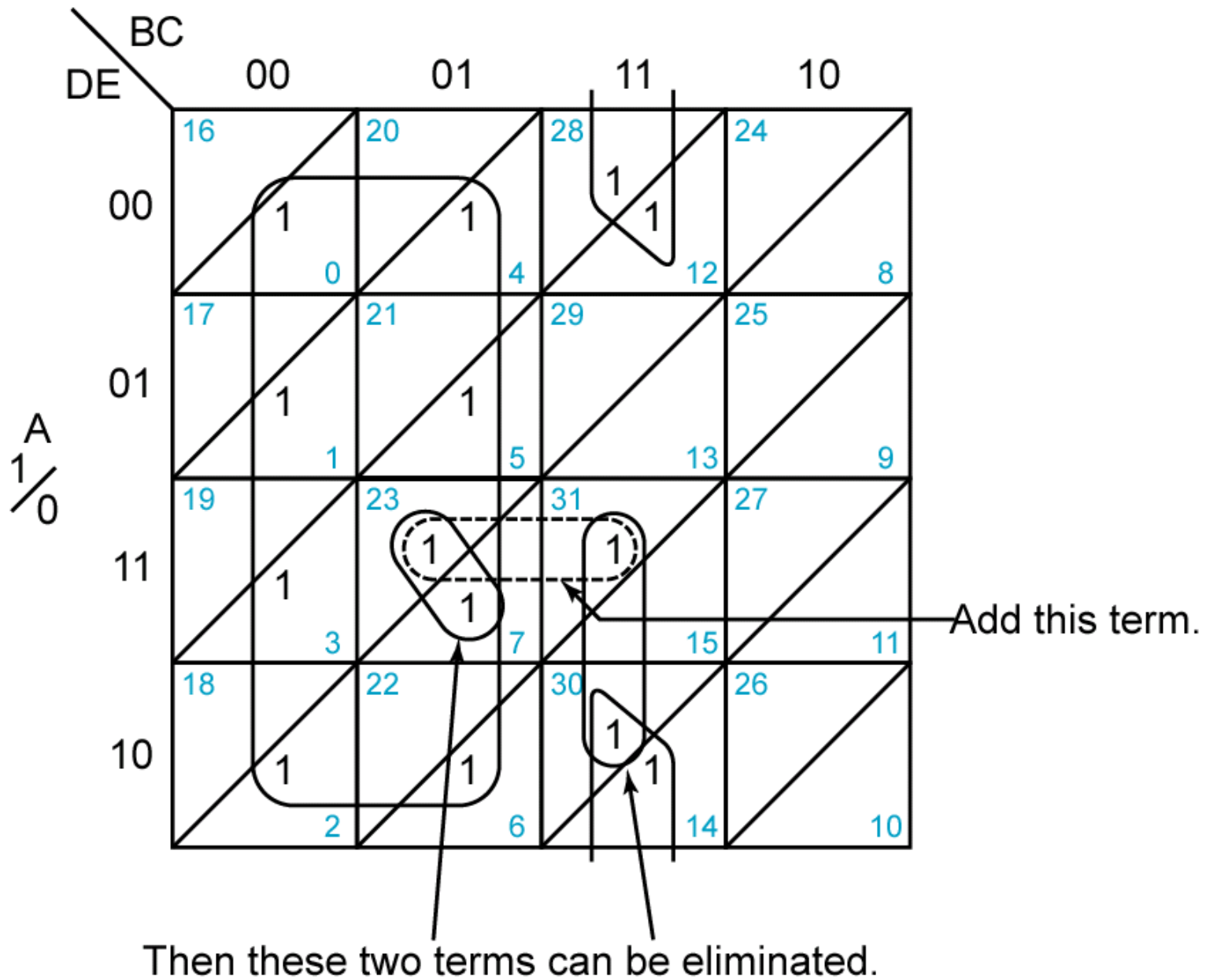


Figure 5-26

Other Forms of Karnaugh Maps

Instead of labeling the sides of a Karnaugh map with 0's and 1's, some people prefer to use Veitch diagrams. In Veitch diagrams, $A = 1$ for the half of the map labeled A, and $A = 0$ for the other half. The other variables have a similar interpretation.

Section 5.7 (p. 146)

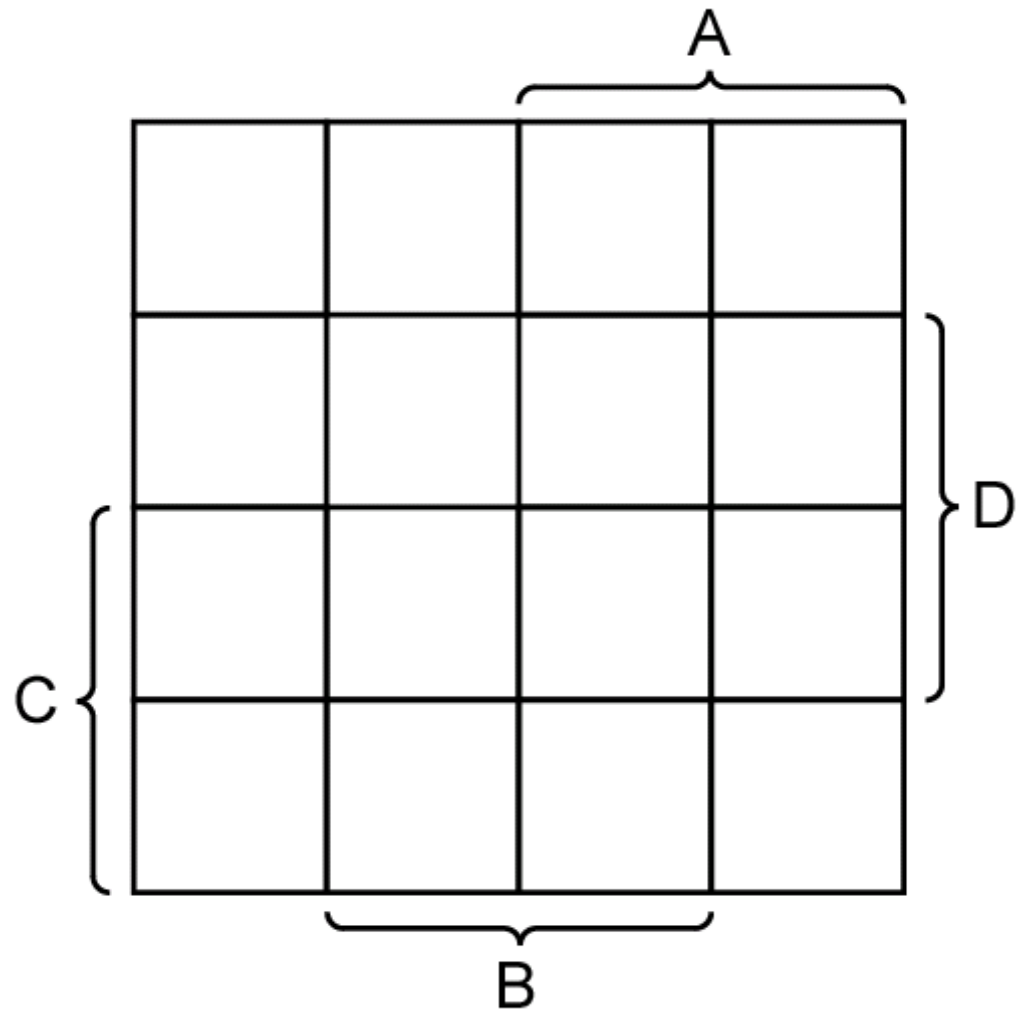
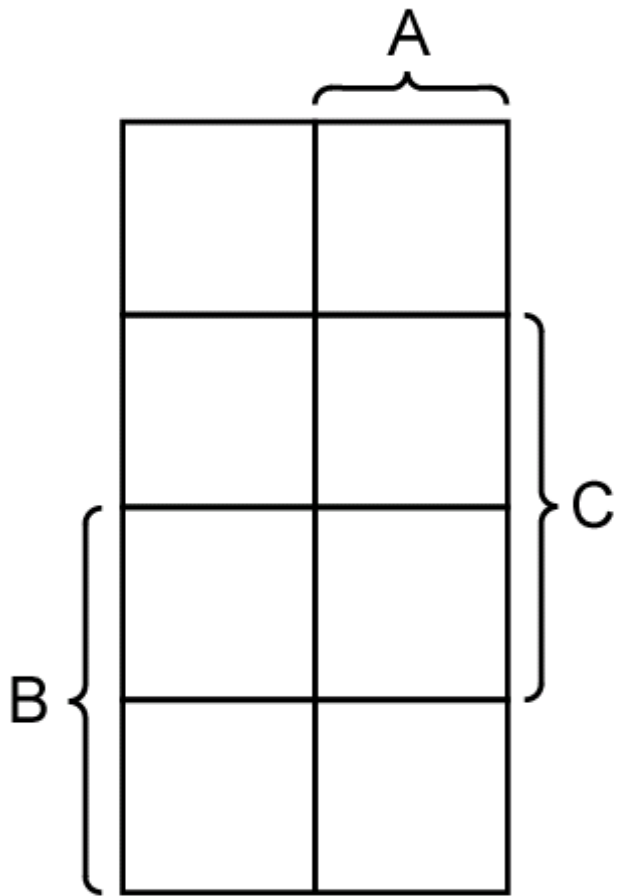
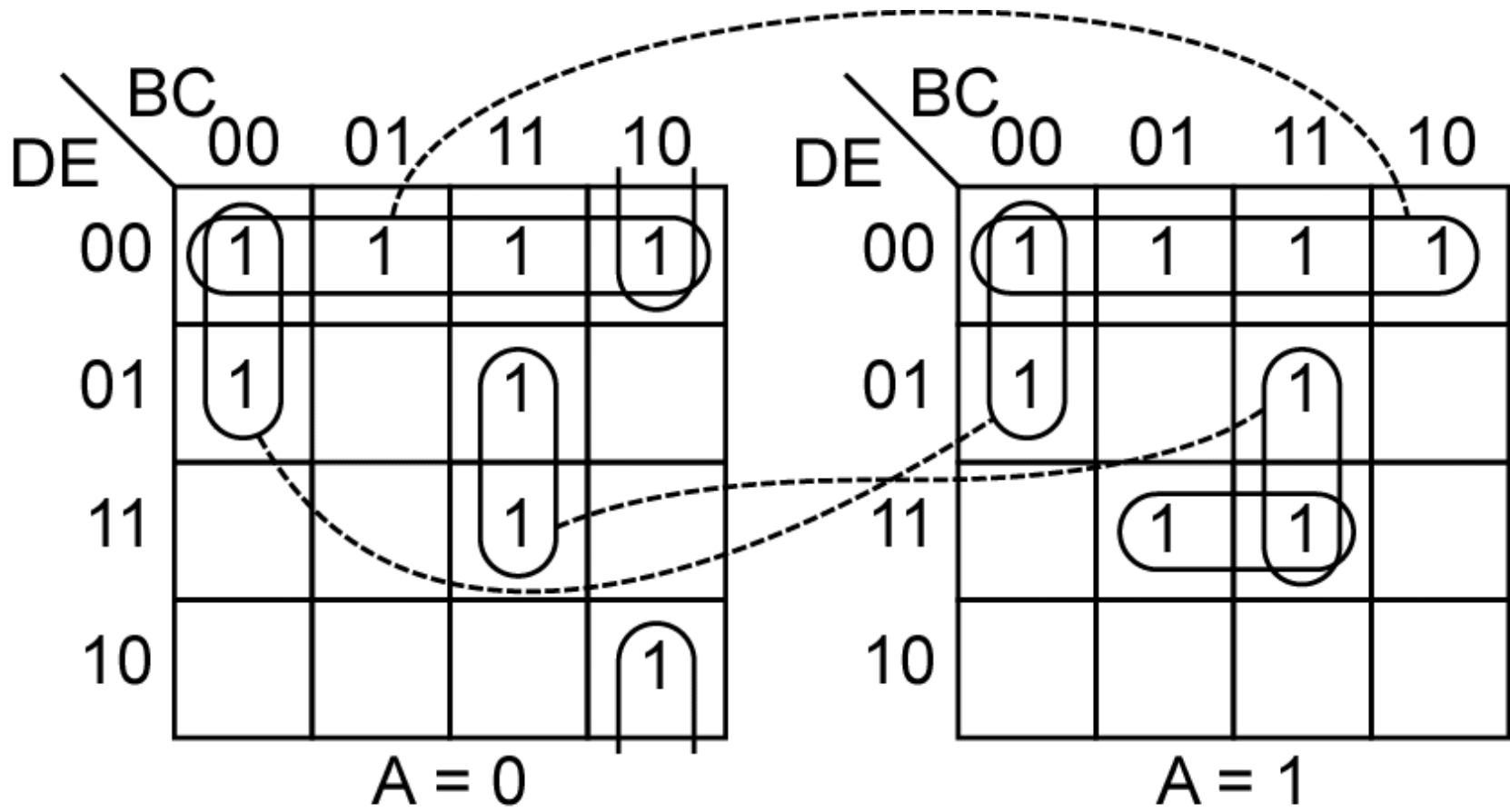


Figure 5-27: Veitch Diagrams

Two alternative forms of five-variable maps are also used. One form simply consists of two four-variable maps side-by-side.

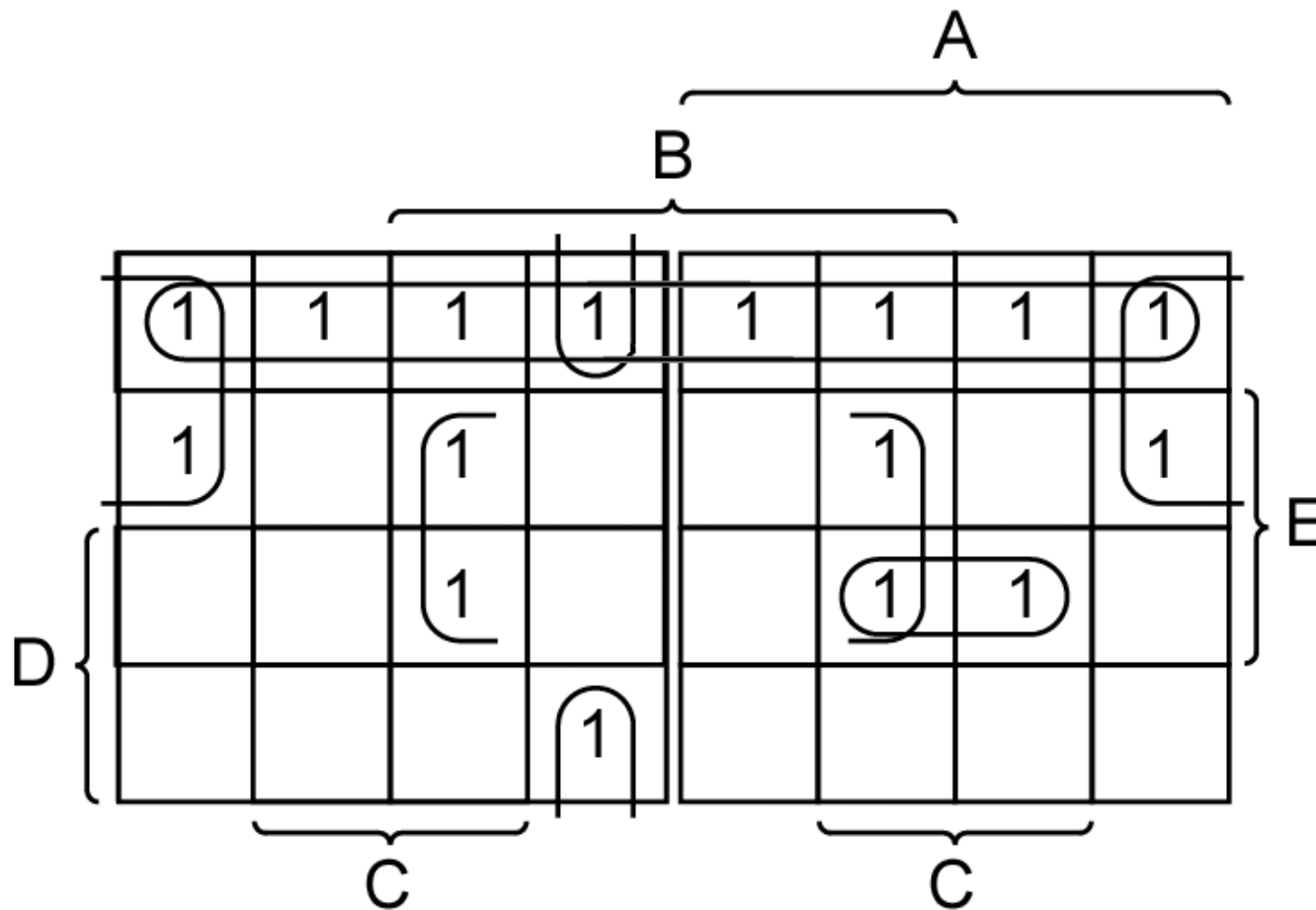
A modification of this uses a *mirror image* map. In this map, first and eighth columns are “adjacent” as are second and seventh columns, third and sixth columns, and fourth and fifth columns.



(a)

$$F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE$$

Figure 5-28: Side-by-side Form of Five-Variable Karnaugh Maps



(b)

$$F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE$$

Figure 5-28: Mirror Image Form of Five-Variable Karnaugh Maps