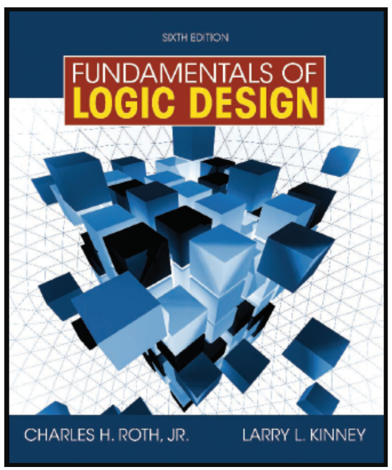
SLIDES FOR

CHAPTER 6

QUINE-McCLUSKEY METHOD



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Determination of Prime Implicants

In order to apply the Quine-McCluskey method to determine a minimum sum-of-products expression for a function, the function must be given as a sum of minterms.

If the function is not in minterm form, the minterm expansion can be found by using one of the techniques given in section 5.3.

Section 6.1 (p. 164)

In the first part of the Quine-McCluskey method, all of the prime implicants of a function are systematically formed by combining minterms.

$$AB'CD' + AB'CD = AB'C$$

$$\underbrace{1\ 0\ 1}_{X}\ \underset{Y}{0} + \underbrace{1\ 0\ 1}_{X}\ \underset{Y'}{1} = \underbrace{1\ 0\ 1}_{X}\ -$$

(the dash indicates a missing variable)

$$A'BC'D + A'BCD' \quad \text{(will not combine)}$$

$$0\ 1\ 0\ 1 + 0\ 1\ 1\ 0 \quad \text{(will not combine)}$$

Equation (6-1)

$$f(a, b, c, d) = \sum m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)$$

To reduce the required number of comparisons, the binary minterms are sorted into groups according to the number of 1's in each term.

Group	Minterm	Binary
group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Equation (6-2)

Table 6-1. Determination of Prime Implicants

	Column I			Column II			Column III	
group 0	0	0000	✓	0, 1	000–	✓	0, 1, 8, 9	–00–
group 1	1	0001	✓	0, 2	00–0	✓	0, 2, 8, 10	–0–0
	2	0010	✓	0, 8	–000	✓	~~0, 8, 1, 9~~	~~–00–~~
	8	1000	✓	1, 5	0–01		~~0, 8, 2,10~~	~~–0–0~~
group 2	5	0101	✓	1, 9	–001	✓	2, 6, 10, 14	- - 1 0
	6	0110	✓	2, 6	0–10	✓	~~2, 10, 6, 14~~	~~- - 1 0~~
	9	1001	✓	2, 10	–010	✓		
	10	1010	✓	8, 9	100–	✓		
group 3	7	0111	✓	8, 10	10–0	✓		
	14	1110	✓	5, 7	01–1			
				6, 7	011–			
				6, 14	–110	✓		
				10, 14	1–10	✓		

Definition: Given a function *F* of *n* variables, a product term *P* is an *implicant* of *F* iff for every combination of values of the *n* variables for which P = 1, *F* is also equal to 1.

Definition: A *prime implicant* of a function *F* is a product term implicant which is no longer an implicant if any literal is deleted from it.

The Prime Implicant Chart

The second part of the Quine-McCluskey method employs a prime implicant chart to select a minimum set of prime implicants.

The minterms of the function are listed across the top of the chart, and the prime implicants are listed down the side.

If a prime implicant covers a given minterm, an X is placed at the intersection of the corresponding row and column.

Section 6.2 (p. 168)

Table 6-2. Prime Implicant Chart

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	×	×					×	⊗		
(0, 2, 8, 10)	$b'd'$	×		×				×		×	
(2, 6, 10, 14)	cd'			×		×				×	⊗
(1, 5)	$a'c'd$		×		×						
(5, 7)	$a'bd$				×		×				
(6, 7)	$a'bc$					×	×				

If a minterm is covered by only one prime implicant, then that prime implicant is called an *essential* prime implicant and must be included in the minimum sum of products.

Table 6-3.

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	*b′c′*	×	×					×	×		
(0, 2, 8, 10)	*b′d′*	×		×				×		×	
(2, 6, 10, 14)	*cd′*			×		×				×	×
(1, 5)	*a′c′d*		×		×						
(5, 7)	*a′bd*				×		×				
(6, 7)	*a′bc*					×	×				

Each time a prime implicant is selected for inclusion in the minimum sum, the corresponding row should be crossed out. After doing this, the columns which correspond to all minterms covered by that prime implicant should also be crossed out.

$$F = \sum m(0, 1, 2, 5, 6, 7)$$

Derivation of prime implicants:

0	000	✓		0, 1	00–
1	001	✓		0, 2	0–0
2	010	✓		1, 5	–01
5	101	✓		2, 6	–10
6	110	✓		5, 7	1–1
7	111	✓		6, 7	11–

Section 6.2 (p. 170)

A *cyclic* prime implicant chart is a prime implicant chart which has two or more X's in every column. We will find a solution by trial and error. We will start by trying (0, 1) to cover column 0.

Table 6-4.

				0	1	2	5	6	7
①	→	(0, 1)	$a'b'$	X	X				
		(0, 2)	$a'c'$	X		X			
		(1, 5)	$b'c$		X		X		
②	→	(2, 6)	bc'			X		X	
③	→	(5, 7)	ac				X		X
		(6, 7)	ab					X	X

We are not guaranteed that the previous solution is minimum. We must go back and solve the problem over again starting with the other prime implicant that covers column 0.

Table 6-5.

			0	1	2	5	6	7
P_1	(0, 1)	$a'b'$	×	×				
P_2	(0, 2)	$a'c'$	×		×			
P_3	(1, 5)	$b'c$		×		×		
P_4	(2, 6)	bc'			×		×	
P_5	(5, 7)	ac				×		×
P_6	(6, 7)	ab					×	×

Petrick's Method

Petrick's method is a technique for determining all minimum sum-of-products solutions from a prime implicant chart.

Petrick's method provides a more systematic way of finding all minimum solutions from a prime implicant chart than the method used previously.

Before applying Petrick's method, all essential prime implicants and the minterms they cover should be removed from the chart.

Section 6.3 (p. 171)

First, we label the rows of the prime implicant chart P_1, P_2, P_3, ect.

We will form a logic function, P, which is true when all of the minterms in the chart have been covered.

Let P_1 be a logic variable which is true when the prime implicant in row P_1 is included in the solution, P_2 be a logic variable which is true when the prime implicant in row P_2 is included in the solution, etc.

Table 6-5.

			0	1	2	5	6	7
P_1	(0, 1)	$a'b'$	×	×				
P_2	(0, 2)	$a'c'$	×		×			
P_3	(1, 5)	$b'c$		×		×		
P_4	(2, 6)	bc'			×		×	
P_5	(5, 7)	ac				×		×
P_6	(6, 7)	ab					×	×

Because column 0 has X's in rows P_1 and P_2, we must choose row P_1 or P_2 in order to cover minterm 0. Therefore, $(P_1 + P_2)$ must be true.

$$P = (P_1 + P_2)(P_1 + P_3)(P_2 + P_4)(P_3 + P_5)(P_4 + P_6)(P_5 + P_6) = 1$$

Section 6.3 (p. 172-173)

$$P = (P_1 + P_2)(P_1 + P_3)(P_2 + P_4)(P_3 + P_5)(P_4 + P_6)(P_5 + P_6) = 1$$

$$P = (P_1 + P_2P_3)(P_4 + P_2P_6)(P_5 + P_3P_6)$$

$$= (P_1P_4 + P_1P_2P_6 + P_2P_3P_4 + P_2P_3P_6)(P_5 + P_3P_6)$$

$$= P_1P_4P_5 + P_1P_2P_5P_6 + P_2P_3P_4P_5 + P_2P_3P_5P_6 + P_1P_3P_4P_6 + P_1P_2P_3P_6 + P_2P_3P_4P_6 + P_2P_3P_6$$

Use $X + XY = X$ to eliminate redundant terms, yielding:

$$P = P_1P_4P_5 + P_1P_2P_5P_6 + P_2P_3P_4P_5 + P_1P_3P_4P_6 + P_2P_3P_6$$

There are two minimal solutions, each with three prime implicants.

In summary, Petrick's method is as follows:

1. Reduce the prime implicant chart by eliminating the essential prime implicant rows and the corresponding columns.
2. Label the rows of the reduced prime implicant chart P_1, P_2, P_3, etc.
3. Form a logic function P which is true when all columns are covered. P consists of a product of sum terms, each sum term having the form $(P_{i0} + P_{i1} + \ldots)$, where P_{i0}, P_{i1} . . . represent the rows which cover column i.

4. Reduce *P* to a minimum sum of products by multiplying out and applying $X + XY = X$.

5. Each term in the result represents a solution, that is, a set of rows which covers all of the minterms in the table. To determine the minimum solutions (as defined in Section 5.1), find those terms which contain a minimum number of variables. Each of these terms represents a solution with a minimum number of prime implicants.

6. For each of the terms found in step 5, count the number of literals in each prime implicant and find the total number of literals. Choose the term or terms which correspond to the minimum total number of literals, and write out the corresponding sums of prime implicants.

Simplification of Incompletely Specified Functions

In this section, we will show how to modify the Quine-McCluskey method in order to obtain a minimum solution when don’t-care terms are present.

Section 6.4 (p. 173)

1	0001	✓
2	0010	✓
3	0011	✓
9	1001	✓
10	1010	✓
7	0111	✓
11	1011	✓
13	1101	✓
15	1111	✓

(1, 3)	00–1	✓
(1, 9)	–001	✓
(2, 3)	001–	✓
(2, 10)	–010	✓
(3, 7)	0–11	✓
(3, 11)	–011	✓
(9, 11)	10–1	✓
(9, 13)	1–01	✓
(10, 11)	101–	✓
(7, 15)	–111	✓
(11, 15)	1–11	✓
(13, 15)	11–1	✓

(1, 3, 9, 11)	–0–1
(2, 3, 10,11)	–01–
(3, 7, 11, 15)	- - 1 1
(9, 11, 13, 15)	1 - - 1

The don't-care terms are treated like required minterms when finding the prime implicants.

F(A, B, C, D) **=**

$$\Sigma\, m(2, 3, 7, 9, 11, 13) + \Sigma\, d(1, 10, 15)$$

Section 6.4 (p. 173)

The don’t-care columns are omitted when forming the prime implicant chart:

	2	3	7	9	11	13
(1, 3, 9, 11)		×		×	×	
*(2, 3, 10, 11)	×	×			×	
*(3, 7, 11, 15)		×	×		×	
*(9, 11, 13, 15)				×	×	×

*indicates an essential prime implicant.

$$F = B'C + CD + AD$$

Section 6.4 (p. 174)

Simplification Using Map-Entered Variables

Although the Quine-McCluskey method can be used with functions with a fairly large number of variables, it is not very efficient for functions that have many variables and relatively few terms.

By using map-entered variables, Karnaugh map techniques can be extended to simplify functions with more than four or five variables.

Section 6.5 (p. 174)

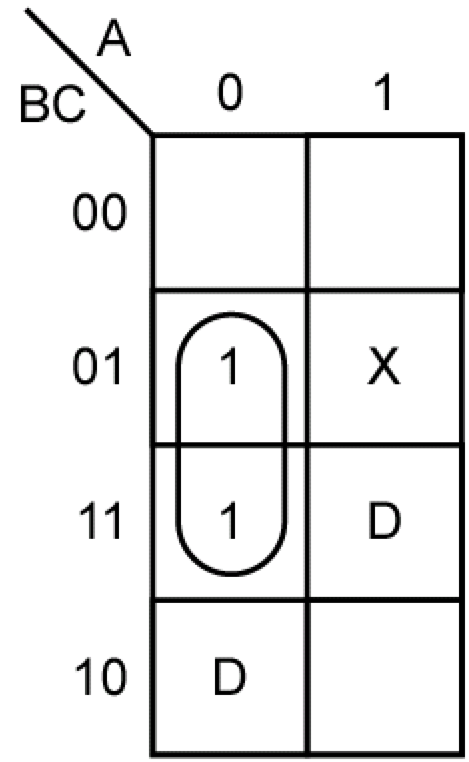
We will simplify

$F(A, B, C, D) = A'B'C + A'BC + A'BC'D + ABCD + (AB'C)$

using a 3-variable map with D as a map-entered variable. Placing D in squares 010 and 111 means that minterms $A'BC'$ and ABC are present when $D = 1$.

To find a minimal expression for F, we will first consider $D = 0$.

When $D = 0$, F reduces to $A'C$.



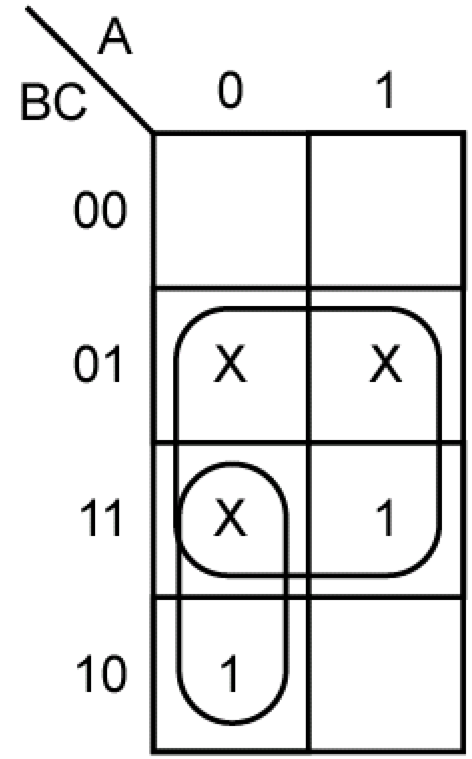
(a)

Figure 6-2: **Simplification Using a Map-Entered Variable**

$F(A, B, C, D) = A'B'C + A'BC + A'BC'D + ABCD + (AB'C)$

Next, consider $D = 1$. Notice the two 1's on the original map have been replaced with don't cares because they have already been covered by $A'C$.

When $D = 1$, F simplifies to $D(C + A'B)$.



(b)

Figure 6-2: **Simplification Using a Map-Entered Variable**

$F(A, B, C, D) = A'B'C + A'BC + A'BC'D + ABCD + (AB'C)$

Thus, our minimum sum-of-products for F is:

$F = A'C + CD + A'BD$.

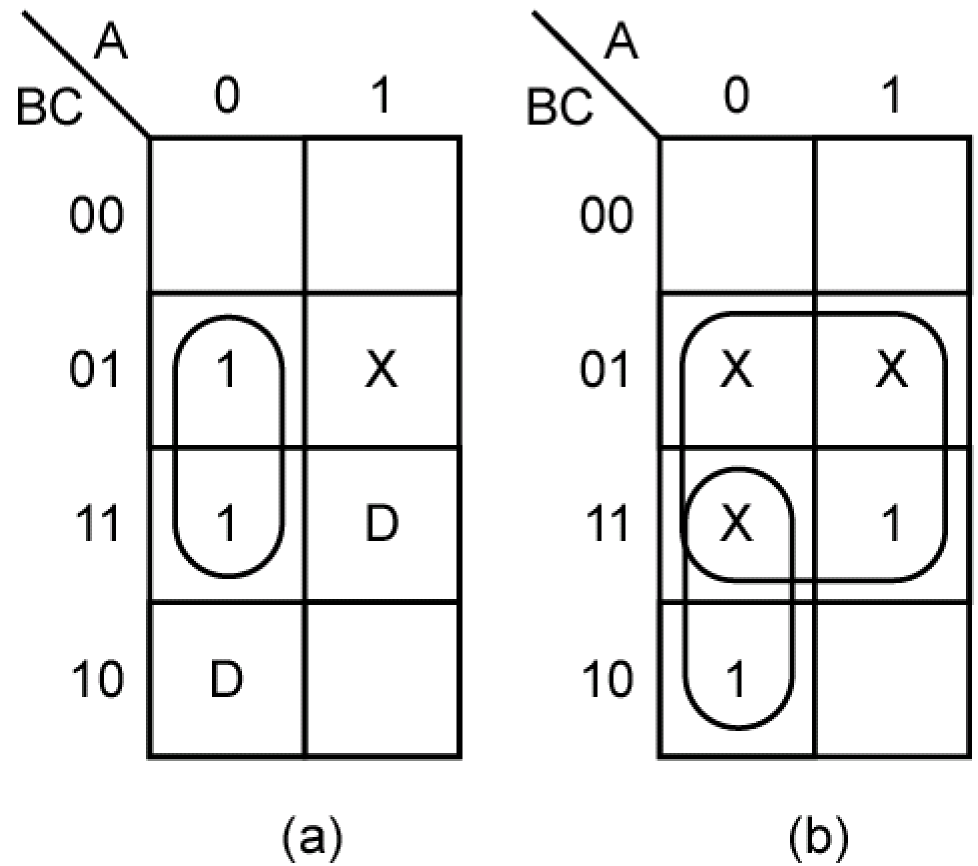


Figure 6-2: **Simplification Using a Map-Entered Variable**

Given a map with variables P_1, P_2, ... Entered into some of the squares, the minimum sum-of-products expression for F can be found as follows:

Find a sum-of-products expression for F of the form

$$F = MS_0 + P_1MS_1 + P_2MS_2 + \cdots$$

where

MS_0 is the minimum sum obtained by setting $P_1 = P_2 = \cdots = 0$.

MS_1 is the minimum sum obtained by setting $P_1 = 1$, $P_j = 0$ ($j \neq 1$), and replacing all 1's on the map with don't-cares.

MS_2 is the minimum sum obtained by setting $P_2 = 1$, $P_j = 0$ ($j \neq 2$), and replacing all 1's on the map with don't-cares.

Next, we will simplify a 6-variable function G(*A, B, C, D, E, F*) using a 4-variable map with map-entered variables E and F.

(a)

CD \ AB	00	01	11	10
00	1			
01	X	E	X	F
11	1	E	1	1
10	1			X

G

Figure 6-1: **Use of Map-Entered Variables**

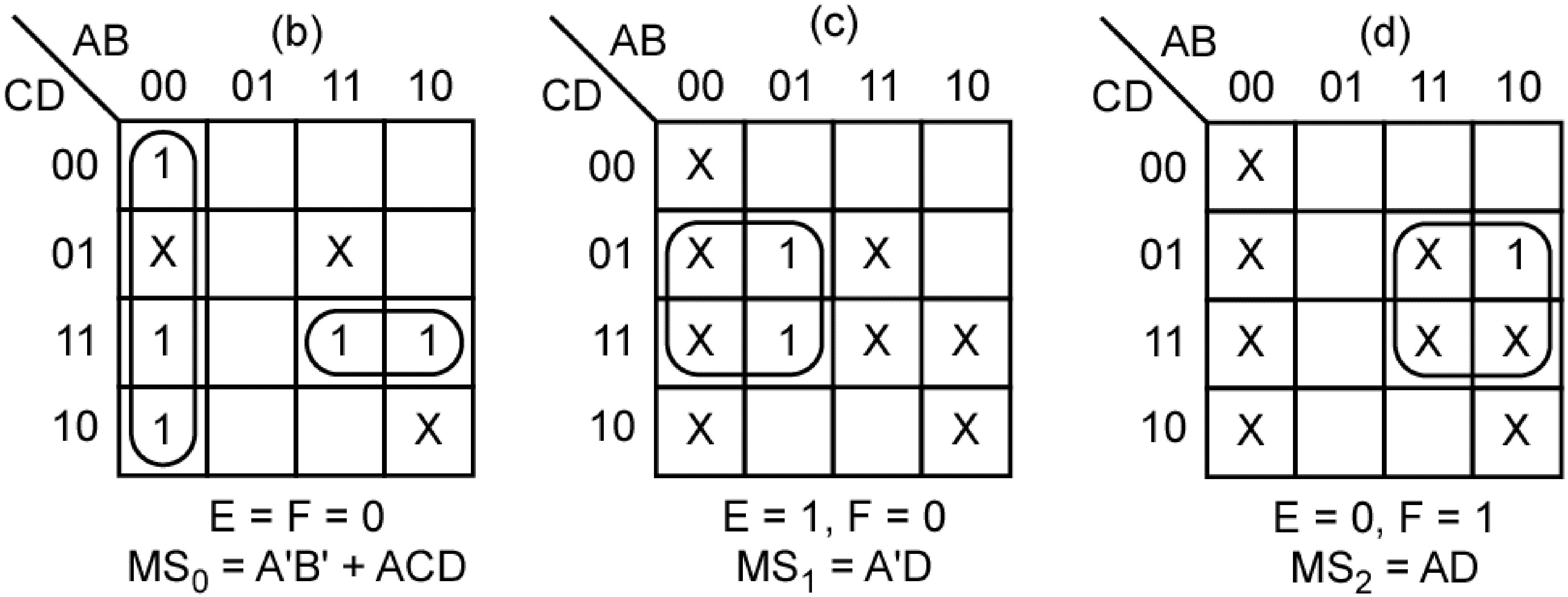


Figure 6-1

The minimum sum of products for *G* is:

$$G = MS_0 + EMS_1 + FMS_2$$
$$G = A'B' + ACD + EA'D + FAD$$